Basic concepts

1. Atmospheric **continuum**
2. Physical dimensions, variables and units
3. Conservation laws (momentum, mass, energy)
4. Eulerian and Lagrangian viewpoints

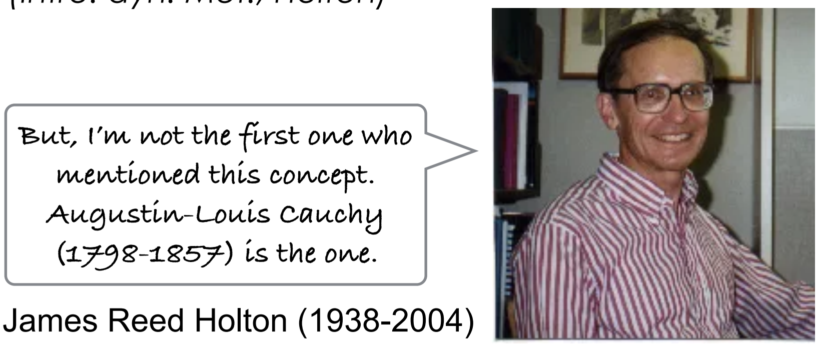
Source: Videezy.com

Concept of a "continuum"

The atmosphere (or fluid) is composed of molecules, but!

"The atmosphere can be regarded as **a continuous fluid medium, or *continuum***. A point in the continuum is regarded as <u>a volume element that is very small</u> compared with the volume of atmosphere under consideration, but still contains a large number of molecules...

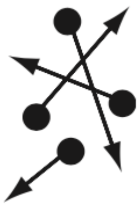
air parcel and air particle are both commonly used to refer to such a point" *(Intro. dyn. Met., Holton)*

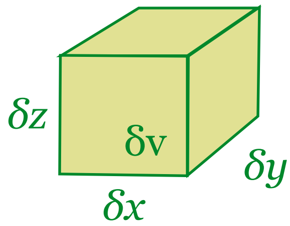


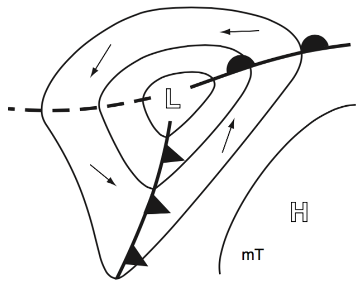
Professor James Reed Holton (1938-2004)

Concept of a "continuum"

Molecules | Control volume | Phenomena of interest







l ~ 68 nm (mean free path under STP) << *$\delta x, \delta y, \delta z$* << *$L$ ~ 5000 km*

Figures from "Practical Meteorology (R. Stull)"

Concept of a "continuum"

"a volume element (control volume, air parcel) contains a **large number** of molecules"

>> **~2.8 × 10^{19} molecules/cm^3**
under standard temperature (273.15 K) and pressure (1013 hPa)

Physical quantities (**pressure, density, temperature, wind**) are **uniquely defined** at each point (air parcel) under the concept of atmospheric continuum.

International system of unit (SI unit)

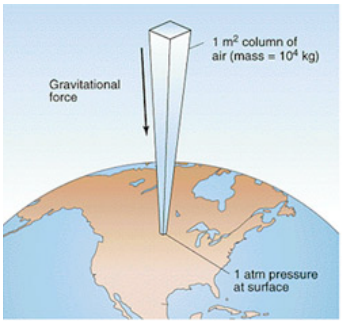
- SI Base Units

Property	Unit	Name
Length	m	Meter
Mass	kg	Kilogram
Time	s	Second
Temperature	K	Kelvin

- Derived Units

Property	Unit	Name
Force	N (kg m s^{-2})	Newton
Pressure	Pa (N m^{-2})	Pascal
Energy	J (N m)	Joule
Power	W (J s^{-1})	Watt
Frequency	Hz (s^{-1})	Hertz

Atmos. variables : Pressure (P)



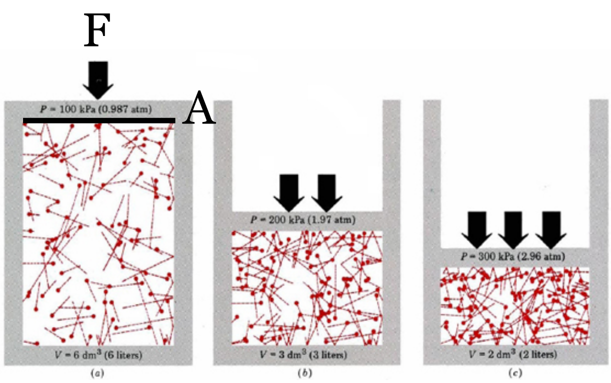
(from faculty.sdmiramar.edu/)

Easy definition (hydrostatic pressure)

Weight of air column
above the measurement point

General definition

Force acting perpendicular to a unit surface ($P = F/A$)
(by randomly moving molecules that bounce off the surface
- Atmospheric pressure)



(from ChemEd DL)

* Units: N/m^2 (Pascal, Pa)

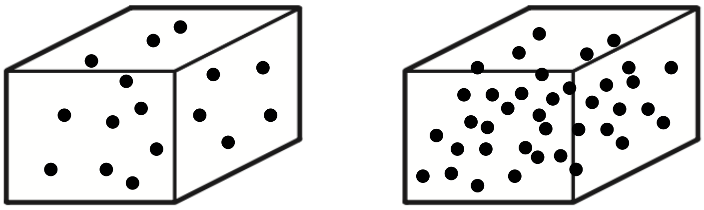
* Standard sea-level pressure:

101325 Pa (1013.25 hPa)

Atmos. variables : Density (ρ)

Density is defined as mass (m) per unit volume (V):

$\rho = m/V$



* Units: kg/m³

* Sea-level density at 288 K (15°C)

ρ_o = 1.225 kg/m³

* Gases such as air are compressible, air density can vary.

* Atmospheric density decreases exponentially with height.

Atmos. variables : Temperature (T)

Simple concept

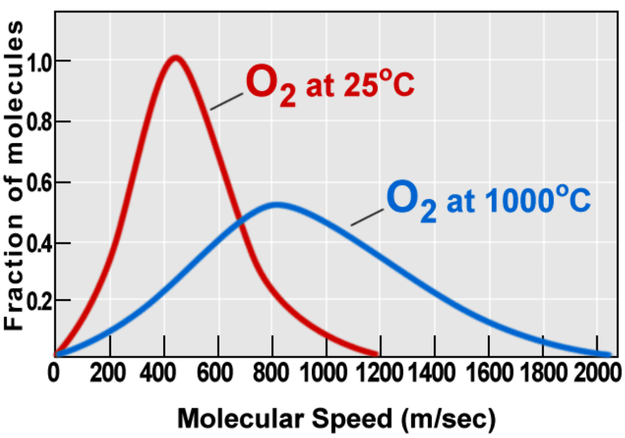
Measure of "hot" and "cold"

Kinematic temperature

Higher temperatures $\boldsymbol{T}$ are associated with greater average molecular speeds $\boldsymbol{v}$

$$N\left[\frac{1}{2}mv^2\right]_{avg} = \frac{3}{2}nR^*T$$

N number of molecules
m mass of molecule (kg)
n amount of molecules (in moles)
R^* gas constant (=8.314 J K⁻¹mol⁻¹)
(N/n = N_A, Avogadro's number)



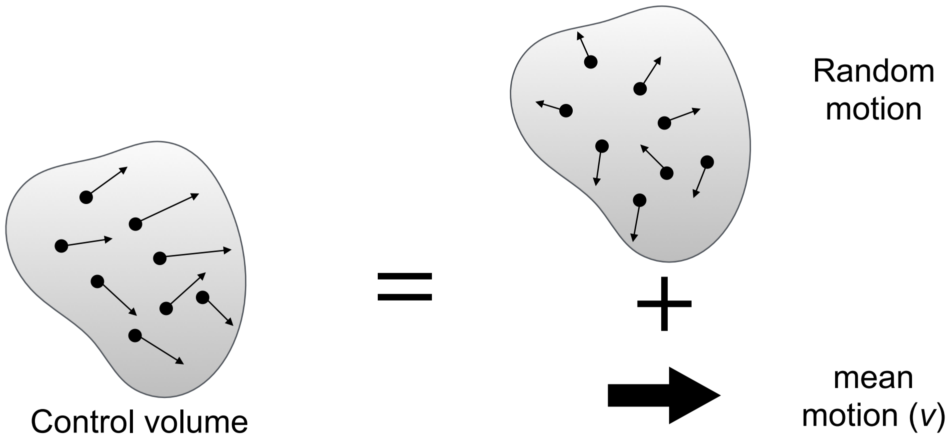
(from www.dlt.ncssm.edu)

Atmos. variables : Wind (v)

Simple concept

"When a group of molecules move predominantly in the same direction, the motion is called **wind**" (Practical Meteorology)

Recall the concept of **continuum**



Note: difference from Kinematic temperature
When molecules move in random directions, the motion is associated with temperature (Practical Meteorology).

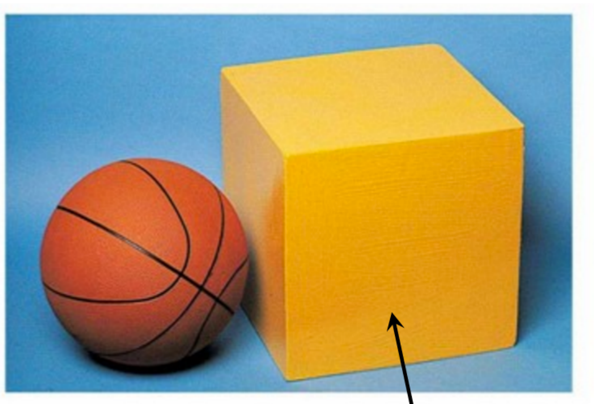
Ideal gas

"An **ideal gas** is a theoretical gas composed of many randomly moving point particles that do not interact except when they collide elastically… At normal conditions, most real gases (N_2, O_2, H_2, etc) behave like an ideal gas" (wikipedia)

Ideal gas law

$$PV = nR^*T$$

P pressure (Pa)
V volume (m^3)
T temperature (K)
n amount of molecules (in moles)
R^* gas constant (=8.314 $J{\cdot}K^{-1}mol^{-1}$)



22.4 L
(22.4×10^{-3} m^3)

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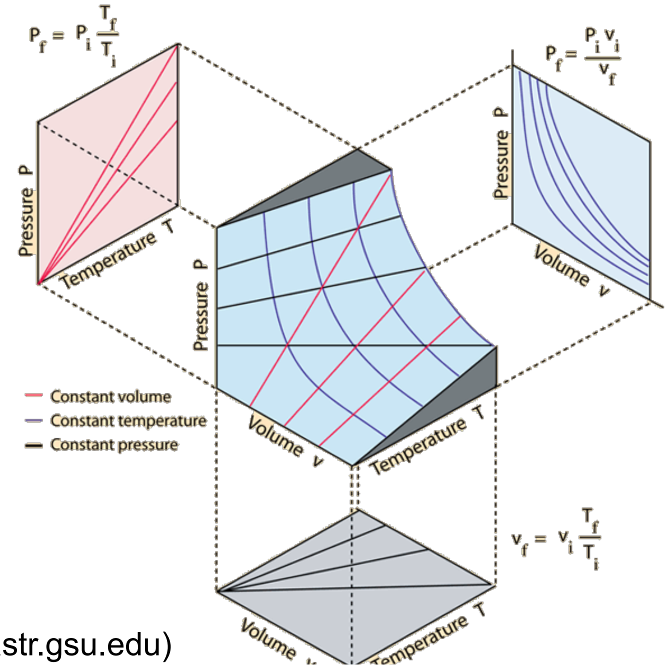

(from hyperphysics.phy-astr.gsu.edu)

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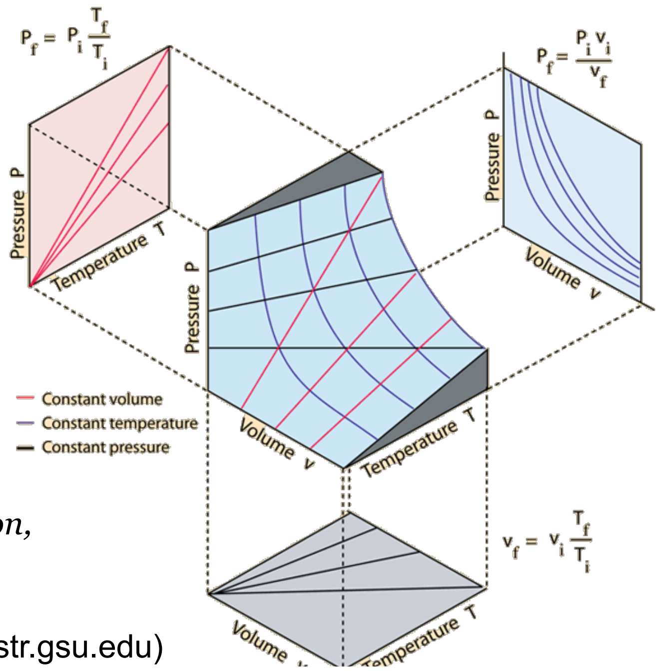


** Under standard temperature (273K) and pressure (1 atm, 1013.25 hPa) condition, 1 mole of ideal gas has volume of 22.4L*

(from hyperphysics.phy-astr.gsu.edu)

Ideal gas law (equation of state)

Atmospheric version

$$P = \rho RT$$

P pressure (Pa)
ρ density (kg m^{-3})
T temperature (K)
R specific gas constant (=287 J K^{-1} kg^{-1}) of dry air - atmospheric version

Derivation

$$PV = nR^*T$$

n amount of molecules (in moles)
R^* gas constant (=8.314 J K^{-1} mol^{-1})
M moler mass of dry air (kg mol^{-1})

$$P = nR^*T/V = nR^*(M/M)T/V = \underbrace{(nM/V)}_{\rho} \times \underbrace{(R^*/M)}_{R} \times T$$

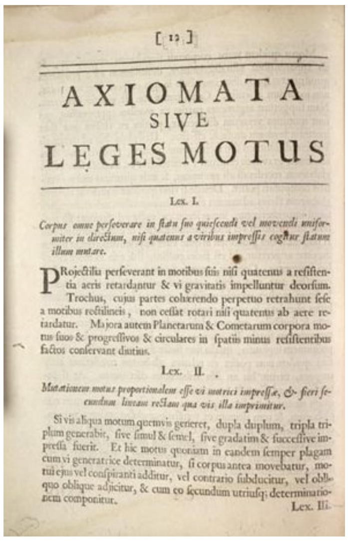
$$P = \rho RT$$

(where $R = R^*/M$)

Conservation of momentum

Newton's law of motion

1. When viewed in an inertial reference frame, an object either **remains** at rest or continues to move **at a constant velocity**, unless acted upon by a force.
2. The vector sum of the forces **F** on an object is equal to the mass *m* of that object multiplied by the acceleration vector **a** of the object: **F = ma**.
3. When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.


[12]

AXIOMATA SIVE LEGES MOTUS

Lex. I.

Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

Projectilia perseverant in motibus suis nisi quatenus a resistentia aeris retardantur & vi gravitatis impelluntur deorsum. Trochus, cujus partes cohærendo perpetuo retrahunt sese a motibus rectilineis, non cessat rotari nisi quatenus ab aere retardatur. Majora autem Planetarum & Cometarum corpora motus suos & progressivos & circulares in spatiis minus resistentibus factos conservant diutius.

Lex. II.

Mutationem motus proportionalem esse vi motrici impressæ, & fieri secundum lineam rectam qua vis illa imprimitur.

Si vis aliqua motum quemvis generet, dupla duplum, tripla triplum generabit, sive simul & semel, sive gradatim & successive impressa fuerit. Et hic motus quoniam in eandem semper plagam cum vi generatrice determinatur, si corpus antea movebatur, motui ejus vel conspiranti additur, vel contrario subducitur, vel oblique adjicitur, & cum eo secundum utriusq; determinationem componitur.

Lex. III.

Principia Mathematica, 1687
(obtained from wikipedia)

Conservation of momentum

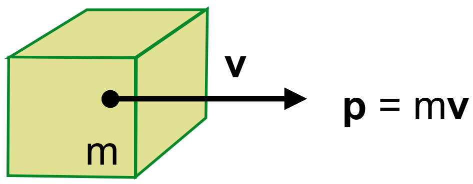
Momentum (p)

The quantity of motion of a moving body, measured as a product of its mass and velocity.

p = m**v** (bold-face means a vector quantity)

* Units: kg m/s (N s)

* **F** = Δ**p**/Δt [**F** = Δ(m**v**)/Δt ➜ **F** = m**a**]



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The quantity of motion of a moving body, measured as a product of its mass and velocity.

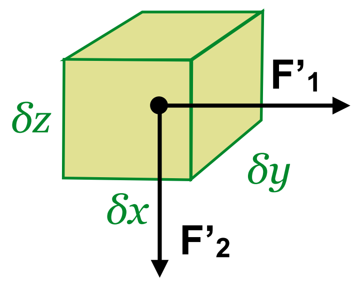
p = m**v** (bold-face means a vector quantity)

* Units: kg m/s (N s)

* $\int_0^t \vec{\mathbf{F}}\, dt = \int_0^{t'} m\vec{\mathbf{a}}\, dt \quad \rightarrow \quad \int_0^t \vec{\mathbf{F}}\, dt = m\vec{\mathbf{v}}\,\Big|_{t=0}^{t=t'} \quad = \quad \vec{\mathbf{p}}\,\Big|_{t=0}^{t=t'}$

Atmospheric version

$$\underbrace{(\rho\delta x\delta y\delta z)}_{m}\underbrace{\frac{D\vec{\mathbf{v}}}{Dt}}_{a\ =}=\underbrace{\vec{\mathbf{F}}_1+\vec{\mathbf{F}}_2+\cdots}_{\Sigma\,\mathbf{F}}$$



$$\frac{D\vec{\mathbf{v}}}{Dt}=\frac{1}{\rho}\frac{\vec{\mathbf{F}}_1+\vec{\mathbf{F}}_2+\cdots}{\delta x\delta y\delta z}=\frac{1}{\rho}\vec{\mathbf{F}}_1{}'+\frac{1}{\rho}\vec{\mathbf{F}}_2{}'+\cdots$$

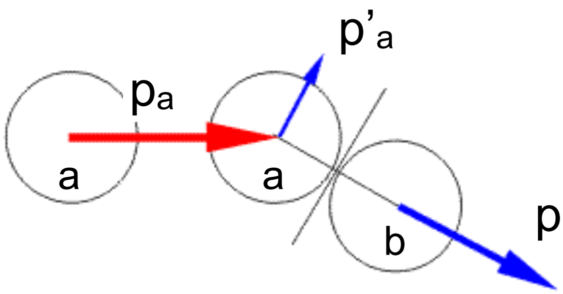
Force per unit volume

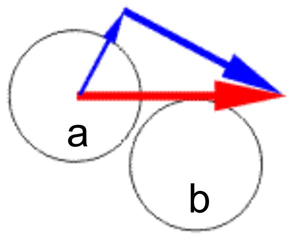
Conservation of momentum

Momentum (p)

The quantity of motion of a moving body, measured as a product of its mass and velocity.

p = m**v** (bold-face means a vector quantity)





(from wikipedia, momentum movie)

$p_a + \cancel{p_b} = p'_a + p'_b$

Conservation of mass

Eulerian view (fixed point)

Rate of mass change in a fixed volume must equal the rate of net mass influx through the walls

$$\frac{\partial(m)}{dt} = \left[-\frac{\partial(\rho u \delta y \delta z)}{\partial x}\left(\frac{\delta x}{2}\right)\right] + \left[\frac{\partial(\rho u \delta y \delta z)}{\partial x}\left(-\frac{\delta x}{2}\right)\right] + \ldots$$

$$= -\frac{\partial(\rho u)}{\partial x}\delta x \delta y \delta z - \frac{\partial(\rho v)}{\partial y}\delta x \delta y \delta z - \frac{\partial(\rho w)}{\partial z}\delta x \delta y \delta z$$

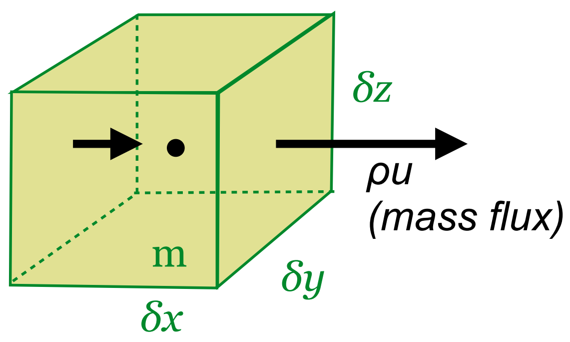
$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial y} - \frac{\partial(\rho w)}{\partial z}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{\mathbf{v}}) = 0}$$

popular form

$m = \rho \delta x \delta y \delta z$



Conservation of energy

First law of thermodynamics

For a closed system,

$\Delta U = Q - W$ (or $\underline{Q} = \underline{\Delta U} + \underline{W}$)

Heat supplied from outside — *change in Internal energy* — *Work done by the system*

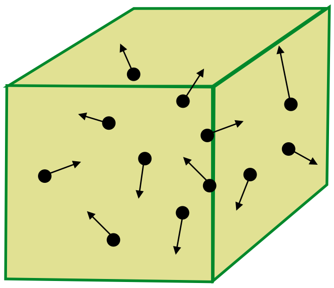
Atmospheric version (ideal gas, per unit mass)

$$c_v \Delta T + p\Delta\alpha = q$$

$$c_v \frac{DT}{Dt} + p\frac{D\alpha}{Dt} = J$$

$$\boxed{c_p \frac{DT}{Dt} - \alpha\frac{Dp}{Dt} = J}$$

popular form



$\Delta U/m = c_v T$
$= (5/2)\, RT$

$c_v = 717.6\ \mathrm{J\ K^{-1}\ kg^{-1}}$
(specific heat at constant volume)

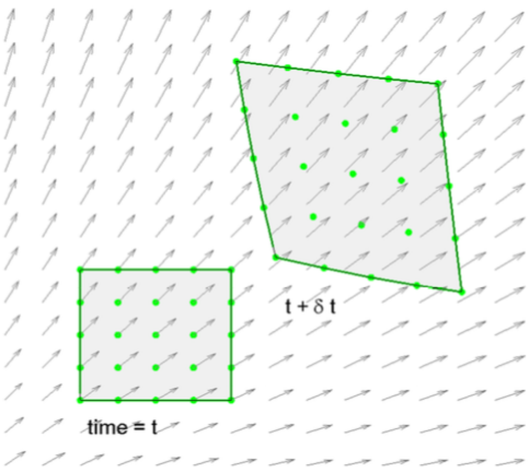
Eulerian vs. Lagrangian

Eulerian description

Describe properties (p,T, ρ,$\mathbf{v}$) of fluid flow **at a fixed point** or volume (easy for math description and measurements)

Lagrangian description

Describe properties (p,T, ρ,$\mathbf{v}$) of fluid flow **following a parcel** or volume (easy to think; '<u>F = ma</u>' works with this concept)



NSF Fluid Mech lecture
(concept ~1.5min, equation ~12min)
https://www.youtube.com/watch?v=mdN8OOkx2ko

J. Price 2006
(Woods Hole Oceanographic Institution)

Material derivative (total derivative)

Material derivative, $\mathrm{D}(\ )/\mathrm{Dt}$

describes the <u>time rate of change</u> of some physical quantity (p,T, ρ,$\mathbf{v}$) for a <u>material element</u> **(Lagrangian)** subjected to a space-and-time-change.

$$\underbrace{\frac{DT}{Dt}}_{\text{Lagrangian}} \equiv \underbrace{\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}}_{\text{Eulerian}}$$

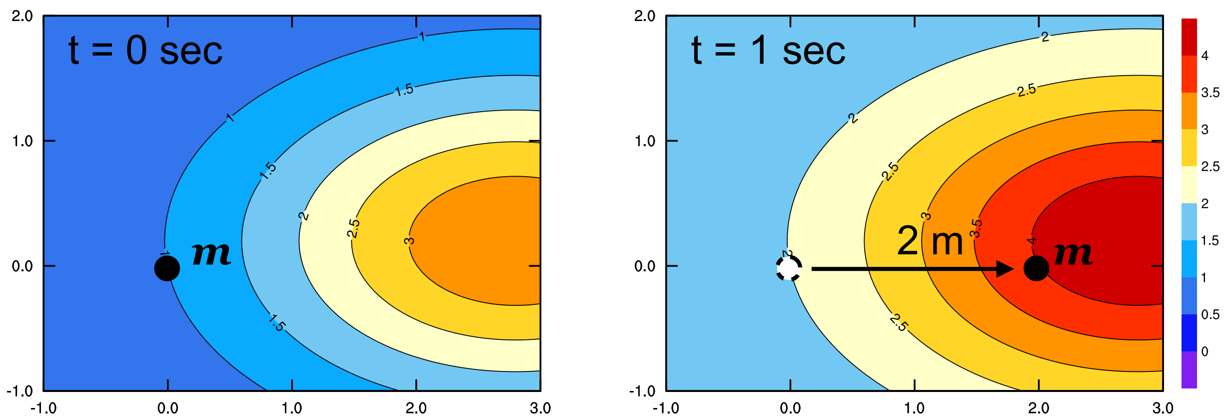
$$\equiv \frac{\partial T}{\partial t} + \vec{\mathbf{v}}\cdot\nabla T$$

$\partial(\ )/\partial t$ partial derivative
*(measures local change, **Eulerian**)*

Material derivative (total derivative)

Material derivative, $\mathrm{D}(\ )/\mathrm{Dt}$

$$\underbrace{\frac{DT}{Dt}}_{3\ \mathrm{K/s}} \equiv \underbrace{\frac{\partial T}{\partial t}}_{1\ \mathrm{K/s}} + \underbrace{u\frac{\partial T}{\partial x}}_{2\ \mathrm{m/s} \times (2\ \mathrm{K})/(2\ \mathrm{m})} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

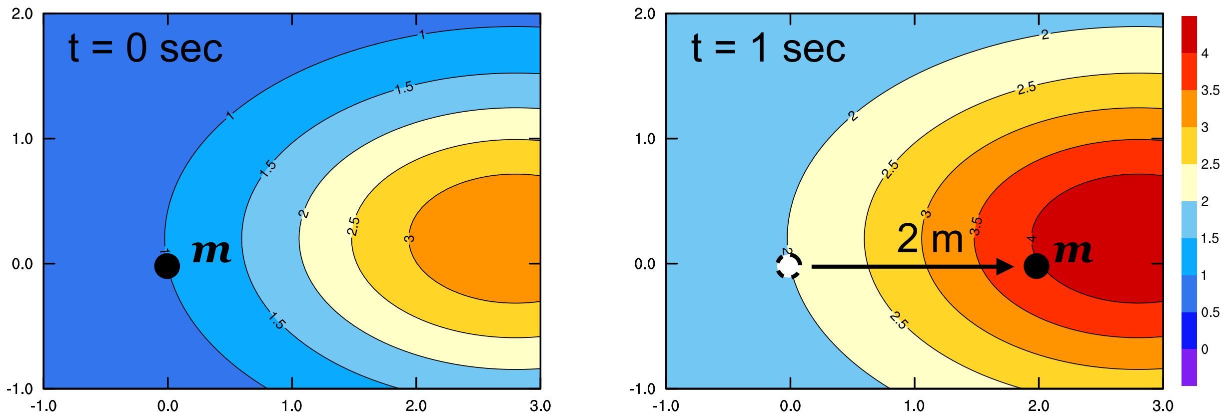


Material derivative (total derivative)

Material derivative, D( )/Dt

$$\frac{DT}{Dt} \equiv \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

3 K/s = 1 K/s + 2 K/s



Conservation of mass

Lagrangian view (following material)

Mass of a material volume should be conserved with motion.

$$\frac{D(m)}{Dt} = 0 \qquad m = \rho\delta x\delta y\delta z$$

$$\frac{D(\rho\delta x\delta y\delta z)}{Dt} = \frac{D(\rho)}{Dt}\delta x\delta y\delta z + \rho\frac{D(\delta x\delta y\delta z)}{Dt} = 0$$

$$\frac{D(\rho)}{Dt} + \rho\frac{1}{\delta x\delta y\delta z}\frac{D(\delta x\delta y\delta z)}{Dt} = 0$$

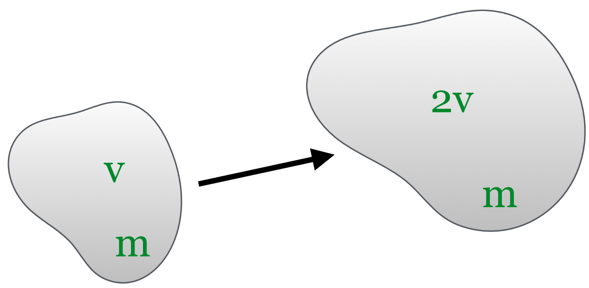
$$\left(\text{using} \quad \lim_{\delta x\to 0}\frac{1}{\delta x}\frac{D\delta x}{Dt} = \frac{\partial u}{\partial x}\right)$$

$$\boxed{\frac{D\rho}{Dt} + \rho\nabla\cdot\vec{\mathbf{v}} = 0}$$

popular form

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{\mathbf{v}}) = 0$$

can change into Eulerian form



References

- Introduction to Dynamic Meteorology (J. R. Holton)
- Practical Meteorology (R. Stull, 2015)
  available in web (https://www.eoas.ubc.ca/books/Practical_Meteorology/)