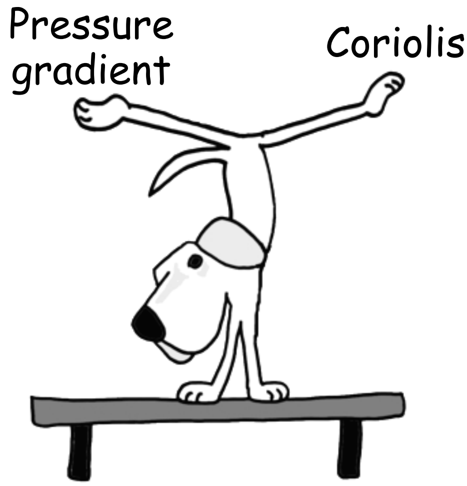
Forces

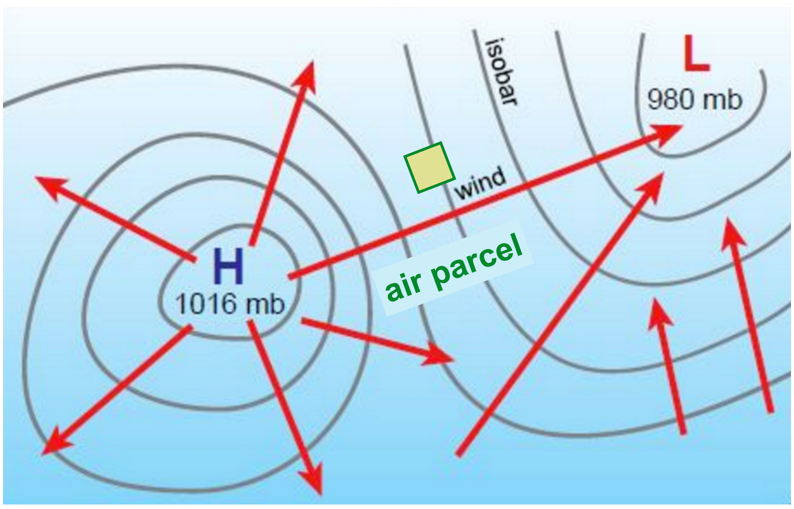
1. Pressure gradient force
2. Gravitational force
3. Viscous force



Pressure gradient force (PGF)

A fluid parcel placed in a **pressure gradient** is subjected to a **net force** associated with pressure difference between one side and the other.

* Recall **pressure** is force per a unit surface, $[N/m^2]$
(by molecules that bounce off the surface, randomly)



(from environment Canada, ec.gc.ca)

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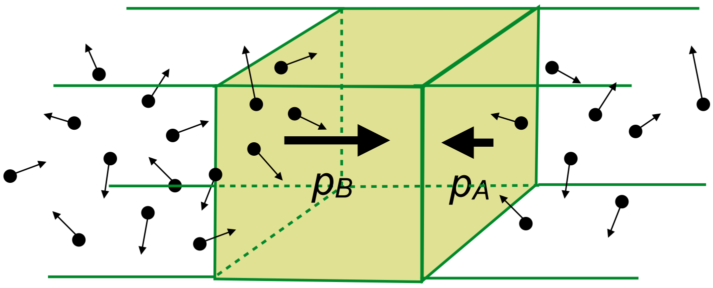


Pressure gradient force (PGF)

A fluid parcel placed in a **pressure gradient** is subjected to a **net force** associated with pressure difference between one side and the other.

$$F_{Ax} = -p_A \delta y \delta z \qquad F_{Bx} = p_B \delta y \delta z$$

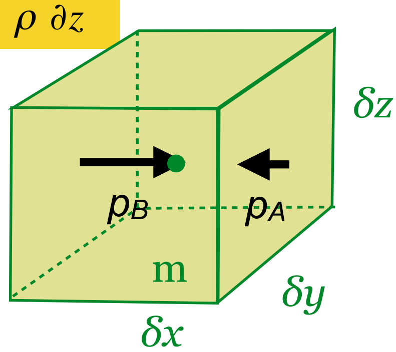
$$\frac{F_x}{m} = -\frac{(p_A - p_B)\delta y \delta z}{\rho \delta x \delta y \delta z}$$

$$\frac{F_x}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad \frac{F_y}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad \frac{F_z}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\mathrm{m} = \rho \delta x \delta y \delta z$$

$$\frac{\vec{\mathbf{F}}_{PGF}}{m} = -\frac{1}{\rho}\nabla p$$

vector form



Gravitational force

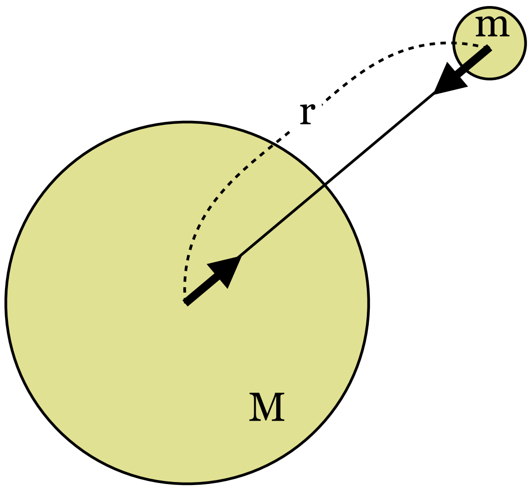
Newton's law of universal gravitation

Any two elements of mass attract each other with a force proportional to their masses and inversely proportional to the square of distance.

$$\vec{\mathbf{F}}_G = -\frac{GMm}{r^2}\hat{\mathbf{r}}$$

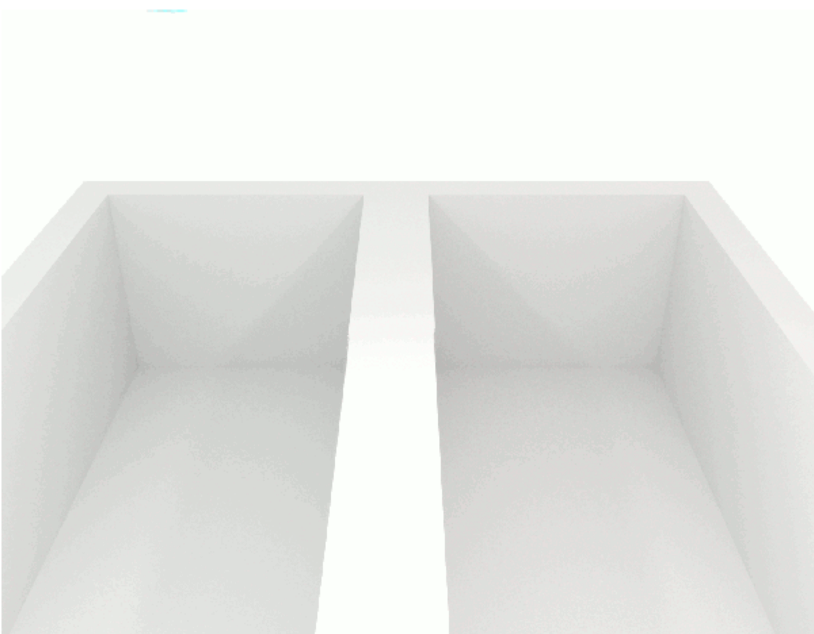
$$\frac{\vec{\mathbf{F}}_G}{m} = -\frac{GM}{r^2}\hat{\mathbf{r}} \approx -\frac{GM}{a^2}\hat{\mathbf{r}} = -\mathbf{k}g_0^*$$

(**k** = unit vector in z direction)



Viscous force

Internal friction in a fluid. Viscosity arises when the fluid velocity varies spatially so that **random molecular (or small-scale eddy) motion** makes a **net transport of momentum** from faster-moving parcel to slower-moving one… *(Intro. dyn. Met., Holton)*.



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$$\frac{F_x}{m} = \frac{(\tau_A - \tau_B)\delta x \delta y}{\rho \delta x \delta y \delta z}$$

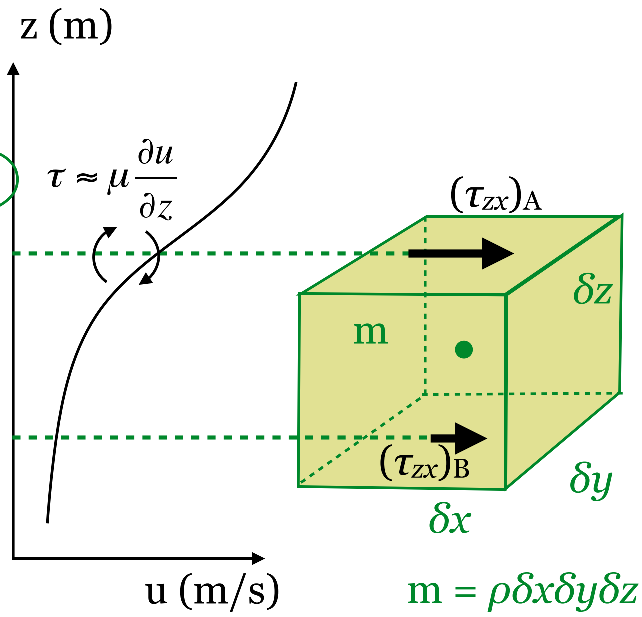
$$\frac{F_x}{m} = \frac{1}{\rho}\frac{\partial \tau}{\partial z}$$

$$\frac{F_x}{m} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right)$$

assuming μ = const

$$\frac{F_x}{m} = \nu \frac{\partial^2 u}{\partial z^2}$$

$$(\nu = \mu / \rho)$$



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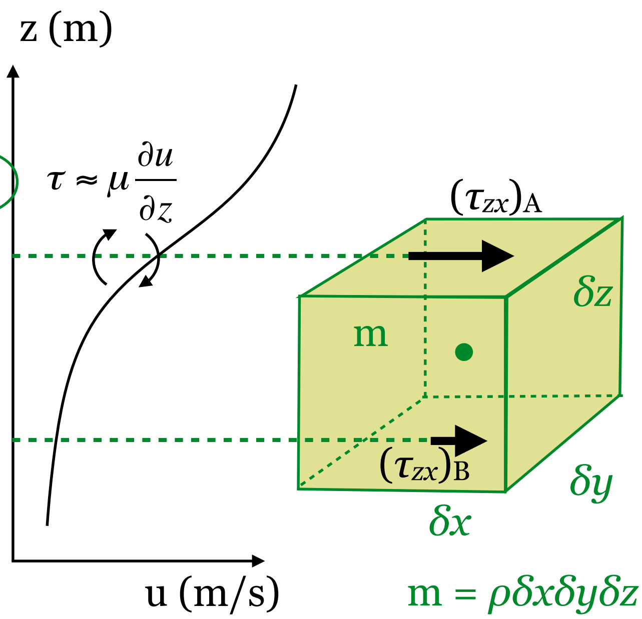
assuming μ = const

$$\frac{F_x}{m} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \nu \nabla^2 u$$

generalized form in x-dir

$$(\nu = \mu / \rho)$$



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In all direction

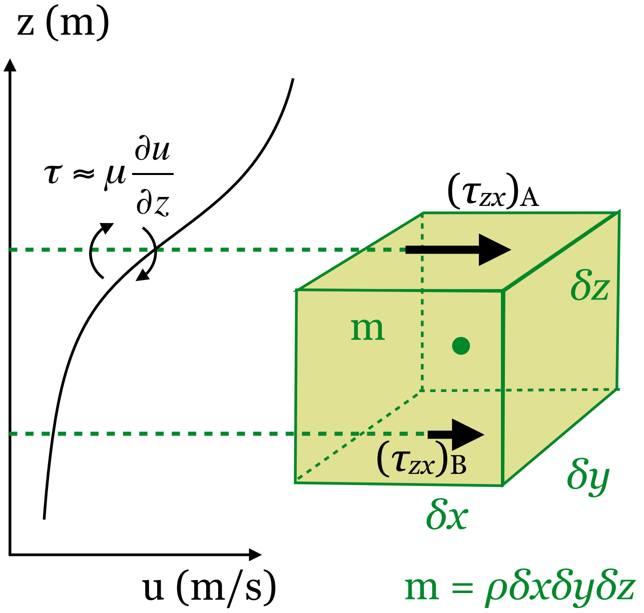
$$\frac{F_x}{m} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \nu\nabla^2 u$$

$$\frac{F_y}{m} = \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \nu\nabla^2 v$$

$$\frac{F_z}{m} = \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \nu\nabla^2 w$$

$$\frac{\vec{\mathbf{F}}_r}{m} = \nu\nabla^2\vec{\mathbf{v}}$$

$$(\nu = \mu/\rho)$$



Viscous force

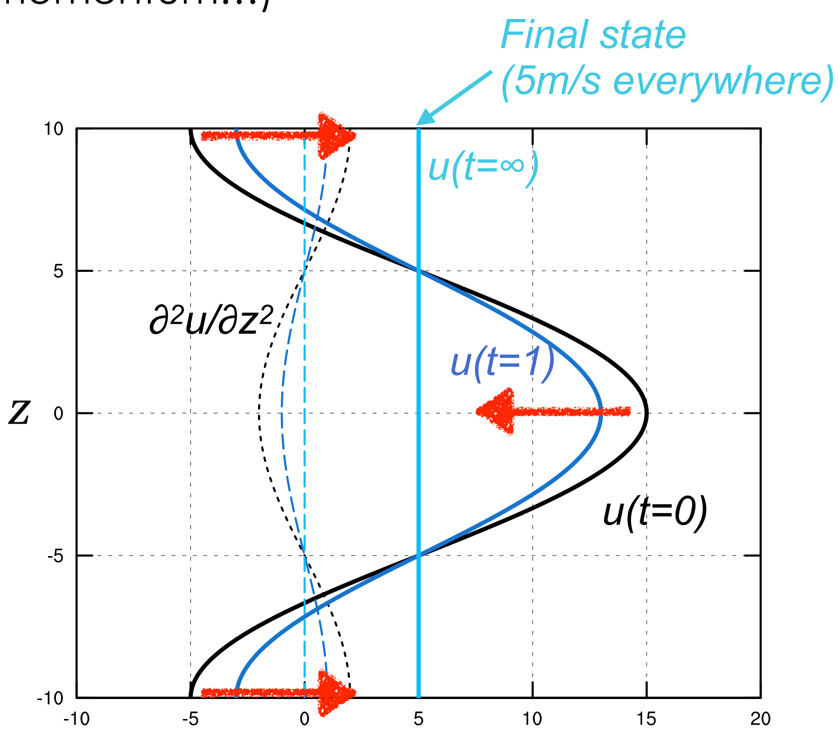
Loss of momentum from maximum value to surrounding region (diffuse out momentum…)

$$\frac{F_x}{m} = \nu\frac{\partial^2 u}{\partial z^2}$$

$$\frac{D\vec{\mathbf{v}}}{Dt} = \frac{\vec{\mathbf{F}}_1}{m} + \frac{\vec{\mathbf{F}}_2}{m} + \cdots$$

$$\frac{du}{dt} = \nu\frac{\partial^2 u}{\partial z^2}$$

$$\frac{du}{dt} = \nu\frac{\partial^2 u}{\partial z^2} \qquad (\propto -\nu u)$$



$$\frac{Du}{Dt} = \nu \frac{\partial^2 u}{\partial z^2}$$

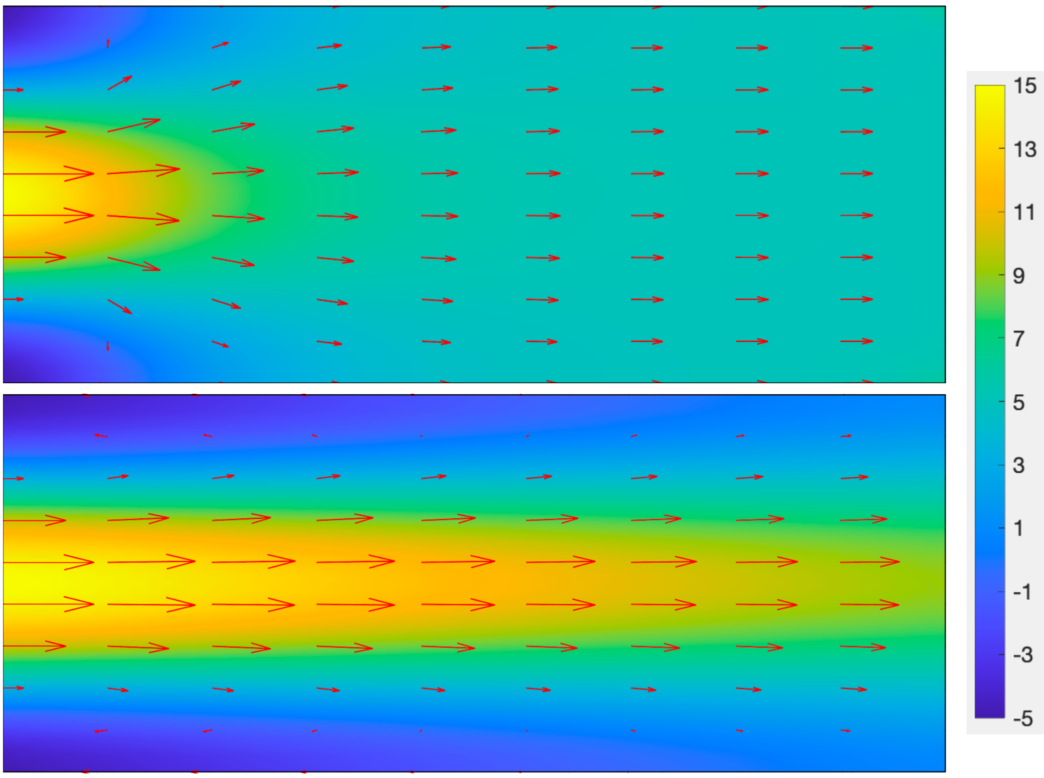
Diffusion equation

$$\frac{Du}{Dt} = \nu \frac{\partial^2 u}{\partial z^2}$$

Diffusion equation

$\nu = 1$

$\nu = 0.1$



Simulated using CFDtool & Matlab