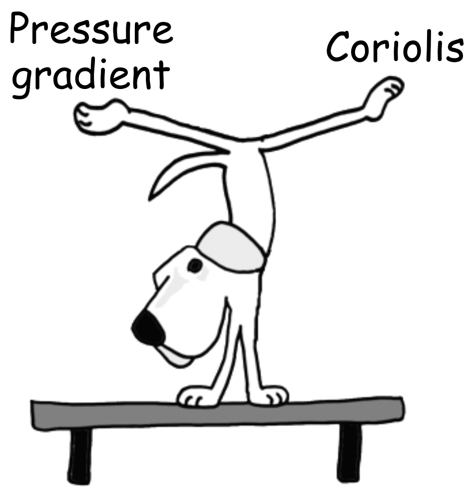
Forces

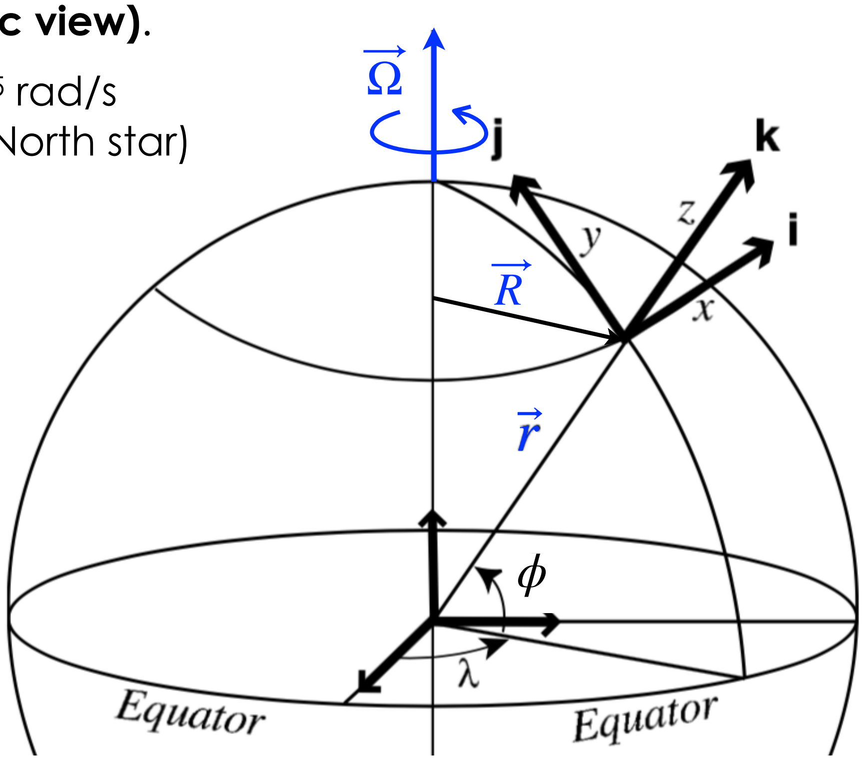
1. Rotating frame
2. Coriolis force
3. Centrifugal force



Rotating Frame

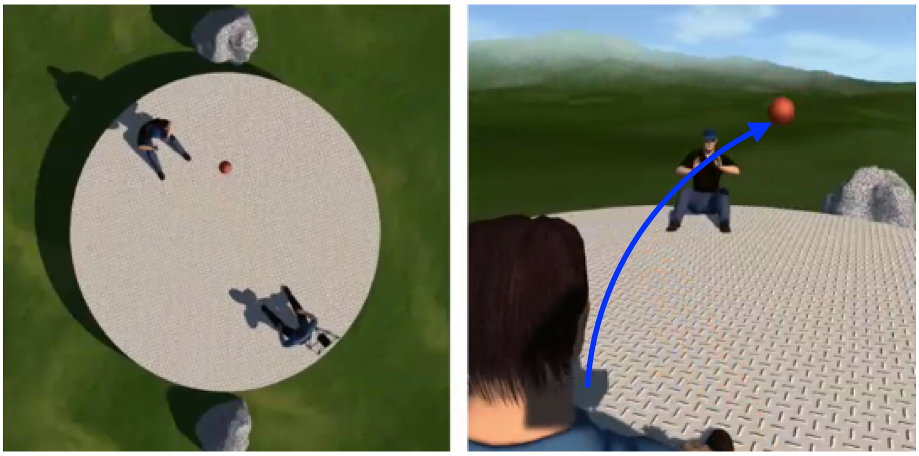
The earth rotates with a period about 24 hours (23h 56m; or 366 rotation per year), **although we can not feel it (human love geocentric view)**.

$\Omega = 7.292 \times 10^{-5}$ rad/s
(directing the North star)

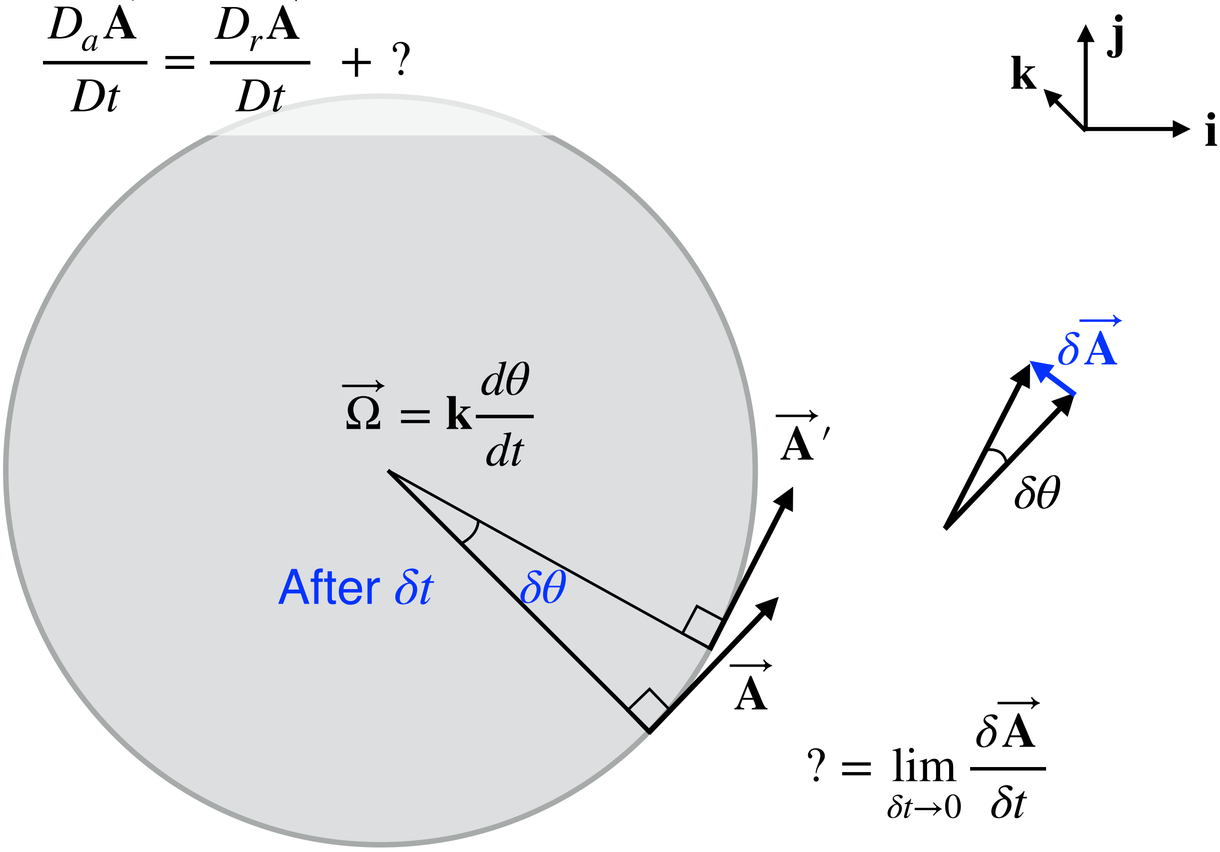


Coriolis force

Newtons' laws of motion work only a inertial frame.
However, we can still use Newtons' law in a rotating frame in a uniform rotation, if we introduce two apparent forces:
1) Coriolis and 2) Centrifugal forces

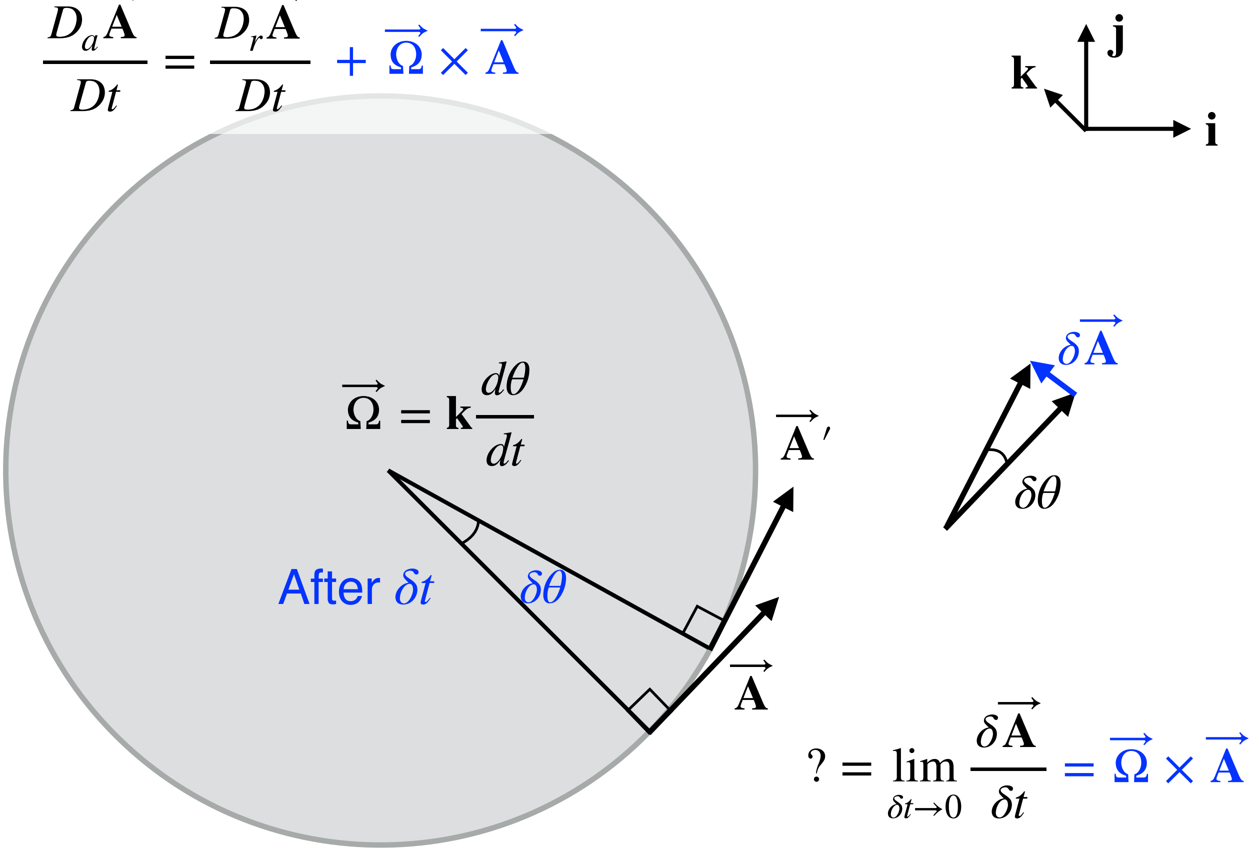


$$\frac{D_a\vec{\mathbf{A}}}{Dt} = \frac{D_r\vec{\mathbf{A}}}{Dt} + ?$$



$$? = \lim_{\delta t \to 0} \frac{\delta\vec{\mathbf{A}}}{\delta t}$$

$$\frac{D_a\vec{\mathbf{A}}}{Dt} = \frac{D_r\vec{\mathbf{A}}}{Dt} + \vec{\boldsymbol{\Omega}} \times \vec{\mathbf{A}}$$



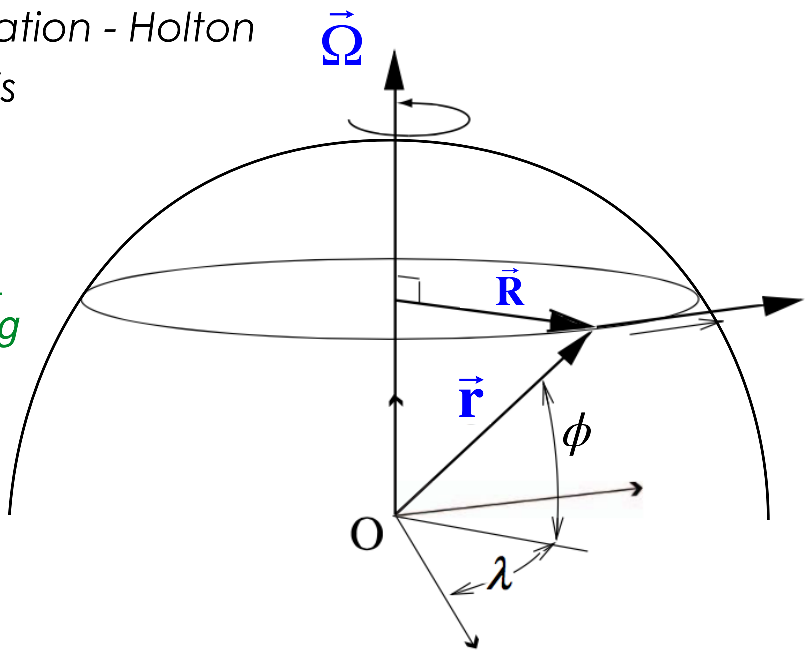
Inertial frame vs. rotation frame

Relationship between total differentiations in
a inertial frame ($D_a\mathbf{A}/Dt$) and a rotating frame ($D_r\mathbf{A}/Dt$)

Can be obtained through
(1) vector calculation - Holton
(2) intuition - Vallis

$$\frac{D_a\vec{\mathbf{A}}}{Dt} = \frac{D_r\vec{\mathbf{A}}}{Dt} + \vec{\boldsymbol{\Omega}} \times \vec{\mathbf{A}}$$

relationship working for any vector ***A***



Velocity (wind)

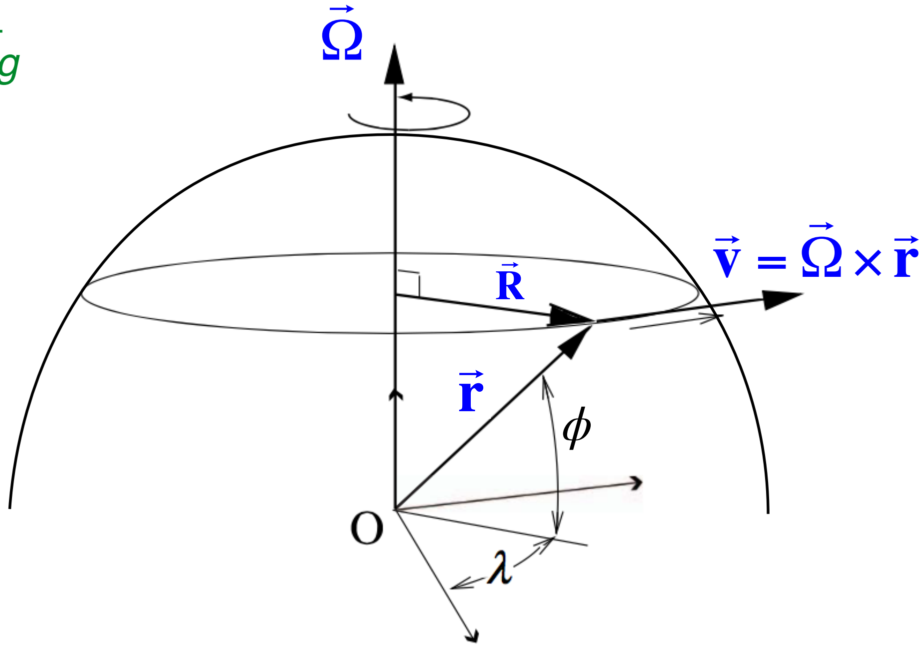
Relationship between total differentiations in
a inertial frame (D_a**A**/Dt) and a rotating frame (D_r**A**/Dt)

$$\frac{D_a\vec{\mathbf{A}}}{Dt} = \frac{D_r\vec{\mathbf{A}}}{Dt} + \vec{\mathbf{\Omega}} \times \vec{\mathbf{A}}$$

relationship working for any vector ***A***

$$\frac{D_a\vec{\mathbf{r}}}{Dt} = \frac{D_r\vec{\mathbf{r}}}{Dt} + \vec{\mathbf{\Omega}} \times \vec{\mathbf{r}}$$

$$\vec{\mathbf{v}}_a = \vec{\mathbf{v}}_r + \vec{\mathbf{\Omega}} \times \vec{\mathbf{r}}$$



Acceleration

Relationship between total differentiations in
a inertial frame (D_a**A**/Dt) and a rotating frame (D_r**A**/Dt)

$$\frac{D_a\vec{\mathbf{A}}}{Dt} = \frac{D_r\vec{\mathbf{A}}}{Dt} + \vec{\mathbf{\Omega}} \times \vec{\mathbf{A}}$$

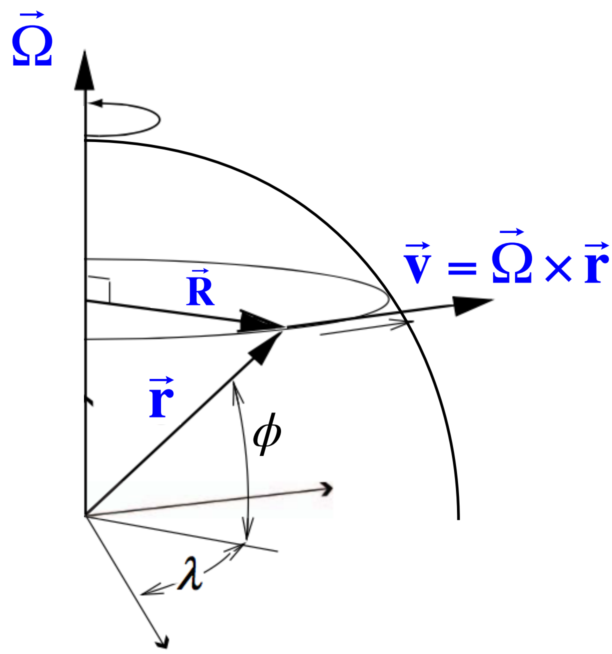
relationship working for any vector ***A***

$$\frac{D_a\vec{\mathbf{v}}_a}{Dt} = \frac{D_r\vec{\mathbf{v}}_a}{Dt} + \vec{\mathbf{\Omega}} \times \vec{\mathbf{v}}_a$$

recall $\vec{\mathbf{v}}_a = \vec{\mathbf{v}}_r + \vec{\mathbf{\Omega}} \times \vec{\mathbf{r}}$

$$\frac{D_a\vec{\mathbf{v}}_a}{Dt} = \frac{D_r\left(\vec{\mathbf{v}}_r + \vec{\mathbf{\Omega}} \times \vec{\mathbf{r}}\right)}{Dt} + \vec{\mathbf{\Omega}} \times \left(\vec{\mathbf{v}}_r + \vec{\mathbf{\Omega}} \times \vec{\mathbf{r}}\right)$$

$$\frac{D_a\vec{\mathbf{v}}_a}{Dt} = \frac{D_r\vec{\mathbf{v}}_r}{Dt} + 2\vec{\mathbf{\Omega}} \times \vec{\mathbf{v}}_r + \vec{\mathbf{\Omega}} \times \left(\vec{\mathbf{\Omega}} \times \vec{\mathbf{r}}\right)$$



Acceleration

$$\frac{D_a\vec{\mathbf{v}}_a}{Dt} = \frac{D_r\vec{\mathbf{v}}_r}{Dt} + \underline{2\vec{\mathbf{\Omega}}\times\vec{\mathbf{v}}_r} + \underline{\vec{\mathbf{\Omega}}\times\left(\vec{\mathbf{\Omega}}\times\vec{\mathbf{r}}\right)}$$

Coriolis force *Centrifugal force* $(= -\Omega^2\vec{\mathbf{R}})$

$$\frac{D_a\vec{\mathbf{v}}_a}{Dt} = -\frac{1}{\rho}\nabla p - \mathbf{k}g^* + \nu\nabla^2\vec{\mathbf{v}}_a$$

*Handle as one term using the concept of **"geoid"** (max of $\Omega^2 R$ is ~0.034 m/s^2; just slightly modify g*➔g)*

$$\frac{D_r\vec{\mathbf{v}}_r}{Dt} = -2\vec{\mathbf{\Omega}}\times\vec{\mathbf{v}}_r + \underline{\Omega^2\vec{\mathbf{R}} - \mathbf{k}g^*} - \frac{1}{\rho}\nabla p + \nu\nabla^2\underline{\vec{\mathbf{v}}_a}$$

↖ viscous force is local cannot feel rotation v_a ➔ v

$$\frac{D\vec{\mathbf{v}}}{Dt} = -2\vec{\mathbf{\Omega}}\times\vec{\mathbf{v}} - \frac{1}{\rho}\nabla p - \mathbf{k}g + \nu\nabla^2\vec{\mathbf{v}}$$

Momentum equation in a rotating frame (with rotation rate Ω)

Equation set

Six equations with six known (u, v, w, T, p, ρ)

Mathematically closed; and **solvable**

num. eq.		
3	$\frac{D\vec{\mathbf{v}}}{Dt} = -\frac{1}{\rho}\nabla p - 2\vec{\mathbf{\Omega}}\times\vec{\mathbf{v}} - \mathbf{k}g + \nu\nabla^2\vec{\mathbf{v}}$	(momentume eq. for u, v, w)
1	$c_p\frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt} = J$	(thermodynamic energy eq. for T)
1	$\frac{D\rho}{Dt} + \rho\nabla\cdot\vec{\mathbf{v}} = 0$	(continuity eq. for ρ)
1	$p = \rho RT$	(equation of state for p)
6		