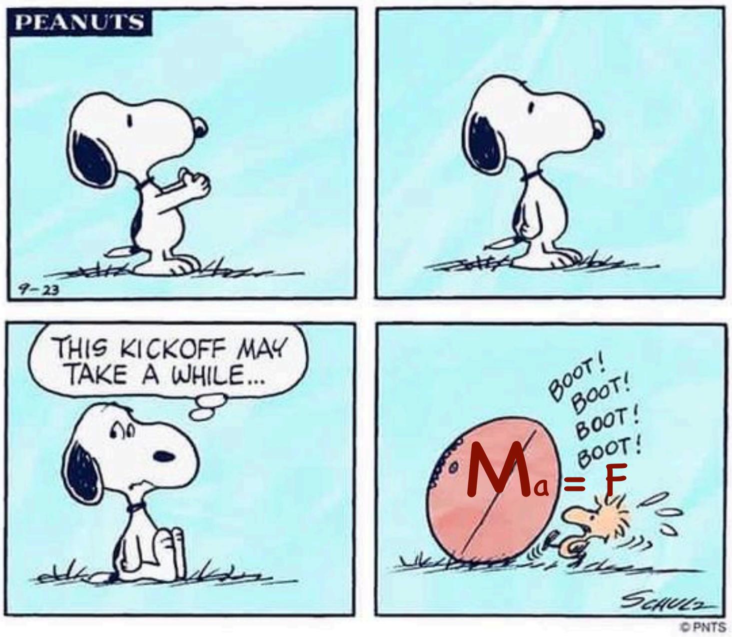
Equations of motion

1. Momentum equations in scalar form
2. Scale analysis
3. Primitive equations



Momentum equation in vector form (with rotation)

$$\frac{D_a \vec{\mathbf{v}}_a}{Dt} = -\frac{1}{\rho}\nabla p - \mathbf{k}g^* + \nu\nabla^2 \vec{\mathbf{v}}_a$$

*Handle as one term using the concept of **"geoid"** (max of $\Omega^2 R$ is ~0.034 m/s^2; just slightly modify $g^* \rightarrow g$)*

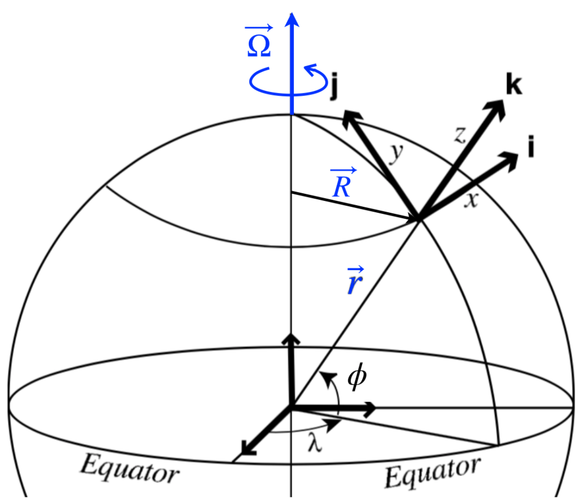
$$\frac{D_r \vec{\mathbf{v}}_r}{Dt} = -2\vec{\mathbf{\Omega}}\times\vec{\mathbf{v}}_r + \underline{\Omega^2\vec{\mathbf{R}} - \mathbf{k}g^*} - \frac{1}{\rho}\nabla p + \nu\nabla^2 \underline{\vec{\mathbf{v}}_a}$$

viscous force is local cannot feel rotation $v_a \rightarrow v$

$$\frac{D\vec{\mathbf{v}}}{Dt} = -2\vec{\mathbf{\Omega}}\times\vec{\mathbf{v}} - \frac{1}{\rho}\nabla p - \mathbf{k}g + \nu\nabla^2\vec{\mathbf{v}}$$

Part 2 **Part 1**

where $\vec{\mathbf{v}} = \mathbf{i}u + \mathbf{j}v + \mathbf{k}w$



Momentum equation in vector form (with rotation)

$$\frac{D\vec{\mathbf{v}}}{Dt} = -2\vec{\boldsymbol{\Omega}} \times \vec{\mathbf{v}} - \frac{1}{\rho}\nabla p - \mathbf{k}g + \nu\nabla^2\vec{\mathbf{v}}$$

where $\vec{\mathbf{v}} = \mathbf{i}u + \mathbf{j}v + \mathbf{k}w$

$$\frac{Du}{Dt} - 2\Omega\sin\phi\ v + 2\Omega\cos\phi\ w + \frac{uw}{r} - \frac{uv\tan\phi}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\frac{Dv}{Dt} + 2\Omega\sin\phi\ u + \frac{vw}{r} + \frac{u^2\tan\phi}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v$$

$$\frac{Dw}{Dt} + -2\Omega\cos\phi\ u - \frac{u^2+v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g - \nu\nabla^2 w$$

Momentum equation in vector form (with rotation)

$$\frac{D\vec{\mathbf{v}}}{Dt} = -2\vec{\boldsymbol{\Omega}} \times \vec{\mathbf{v}} - \frac{1}{\rho}\nabla p - \mathbf{k}g + \nu\nabla^2\vec{\mathbf{v}}$$

where $\vec{\mathbf{v}} = \mathbf{i}u + \mathbf{j}v + \mathbf{k}w$

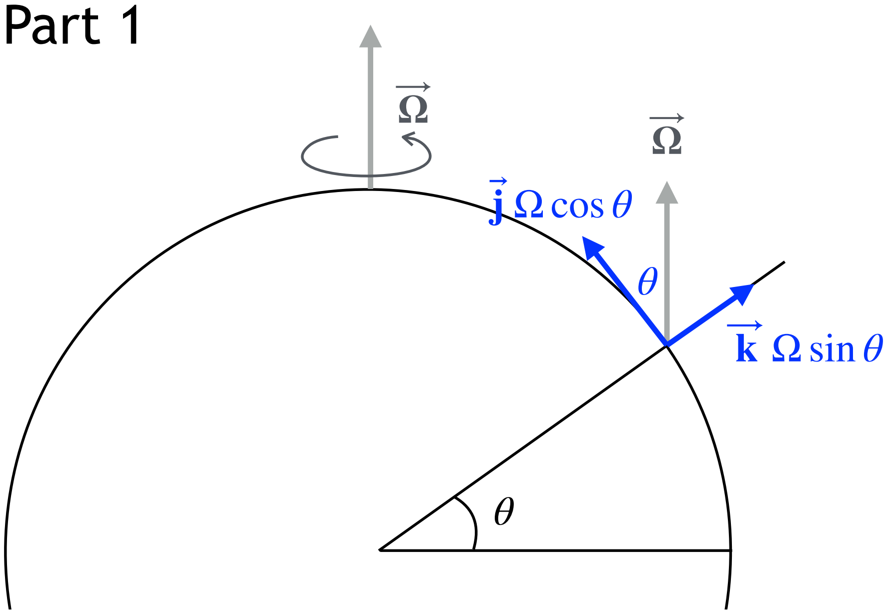
$$\frac{Du}{Dt} - 2\Omega\sin\phi\ v + 2\Omega\cos\phi\ w + \frac{uw}{r} - \frac{uv\tan\phi}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\frac{Dv}{Dt} + 2\Omega\sin\phi\ u + \frac{vw}{r} + \frac{u^2\tan\phi}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v$$

$$\frac{Dw}{Dt} + -2\Omega\cos\phi\ u - \frac{u^2+v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g - \nu\nabla^2 w$$

Part 1 **Part 2**

Part 1



$$2\vec{\Omega}\times\vec{\mathbf{v}}$$

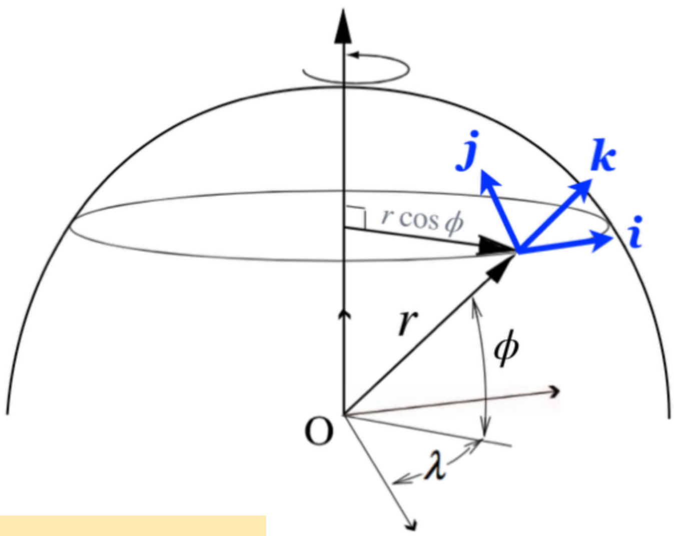
$$= (\mathbf{j}\;\underbrace{2\Omega\cos\theta}_{f^*} + \mathbf{k}\;\underbrace{2\Omega\sin\theta}_{f}) \times (\mathbf{i}\,u + \mathbf{j}\,v + \mathbf{k}\,w)$$

f^* $\quad$ f = Coriolis parameter

Part 2

In spherical coordinates, things we need are

$\frac{\partial i}{\partial x}$	$\frac{\partial i}{\partial y}$	$\frac{\partial i}{\partial z}$
$\frac{\partial j}{\partial x}$	$\frac{\partial j}{\partial y}$	$\frac{\partial j}{\partial z}$
$\frac{\partial k}{\partial x}$	$\frac{\partial k}{\partial y}$	$\frac{\partial k}{\partial z}$



i ***(west-to-east)***

$$\frac{\partial \mathbf{i}}{\partial x} = \frac{\partial \mathbf{i}}{r\cos\phi\,\partial\lambda} = \frac{1}{r\cos\phi}\left(\mathbf{j}\sin\phi - \mathbf{k}\cos\phi\right)$$

$$\frac{\partial \mathbf{i}}{\partial y} = 0$$

$$\frac{\partial \mathbf{i}}{\partial z} = 0$$

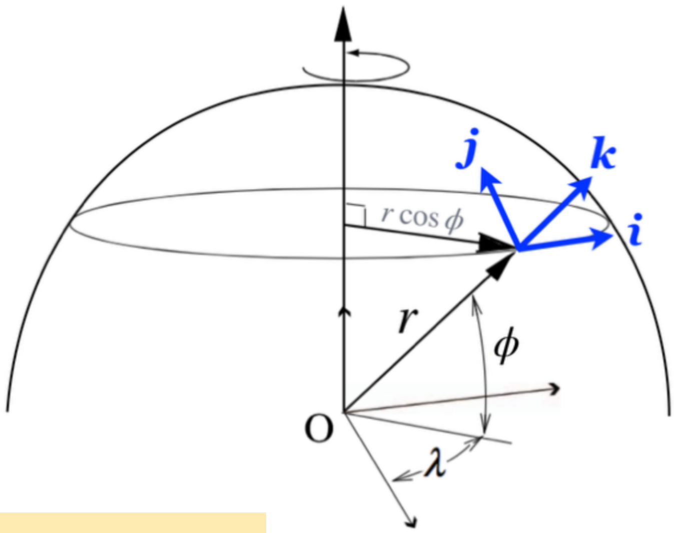
Part 2

In spherical coordinates, things we need are

$$\frac{\partial i}{\partial x} \quad \frac{\partial i}{\partial y} \quad \frac{\partial i}{\partial z}$$

$$\frac{\partial j}{\partial x} \quad \frac{\partial j}{\partial y} \quad \frac{\partial j}{\partial z}$$

$$\frac{\partial k}{\partial x} \quad \frac{\partial k}{\partial y} \quad \frac{\partial k}{\partial z}$$



j ***(south-to-north)***

$$\frac{\partial \mathbf{j}}{\partial x} = \frac{\partial \mathbf{j}}{r\cos\phi\,\partial\lambda} = \frac{1}{r\cos\phi}\left(-\mathbf{i}\sin\phi\right) = -\,\mathbf{i}\frac{\tan\phi}{r}$$

$$\frac{\partial \mathbf{j}}{\partial y} = \frac{\partial \mathbf{j}}{r\partial\phi} = -\,\mathbf{k}\frac{1}{r}$$

$$\frac{\partial \mathbf{j}}{\partial z} = 0$$

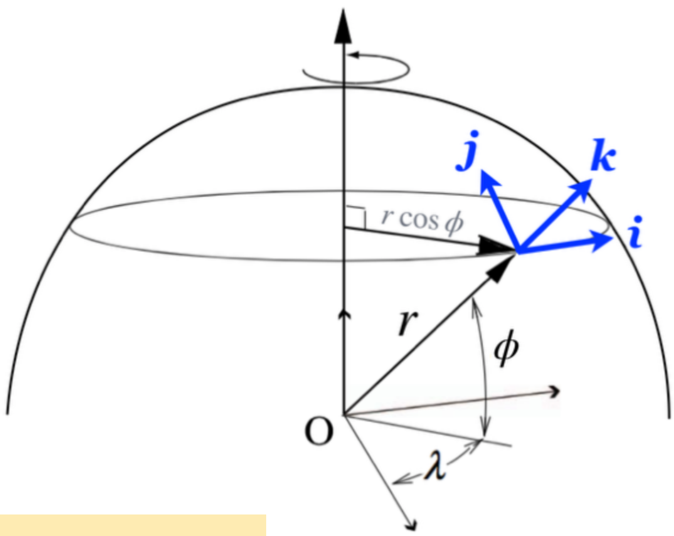
Part 2

In spherical coordinates, things we need are

$$\frac{\partial i}{\partial x} \quad \frac{\partial i}{\partial y} \quad \frac{\partial i}{\partial z}$$

$$\frac{\partial j}{\partial x} \quad \frac{\partial j}{\partial y} \quad \frac{\partial j}{\partial z}$$

$$\frac{\partial k}{\partial x} \quad \frac{\partial k}{\partial y} \quad \frac{\partial k}{\partial z}$$



k ***(bottom-to-top)***

$$\frac{\partial \mathbf{k}}{\partial x} = \frac{\partial \mathbf{k}}{r\cos\phi\,\partial\lambda} = \frac{\mathbf{i}\cos\phi}{r\cos\phi} = \mathbf{i}\frac{1}{r}$$

$$\frac{\partial \mathbf{k}}{\partial y} = \frac{\partial \mathbf{k}}{r\partial\phi} = \mathbf{j}\frac{1}{r}$$

$$\frac{\partial \mathbf{k}}{\partial z} = 0$$