원시방정식계
(Primitive equations)

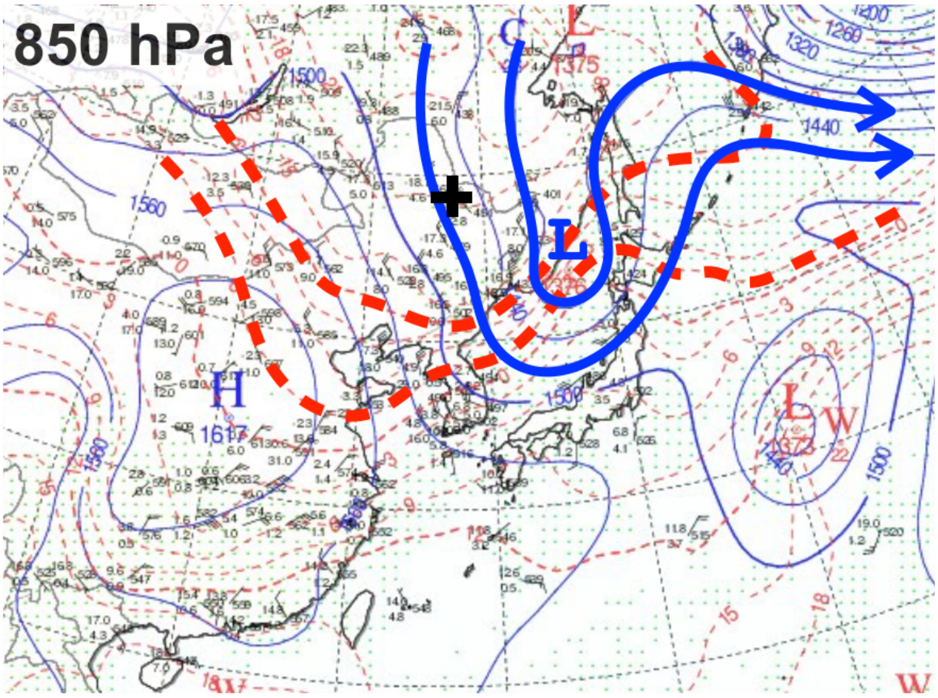
Momentum equation in vector form (with rotation)

$$\frac{Du}{Dt} - 2\Omega\sin\phi\ v + 2\Omega\cos\phi\ w + \frac{uw}{r} - \frac{uv\tan\phi}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\frac{Dv}{Dt} + 2\Omega\sin\phi\ u \qquad\qquad + \frac{vw}{r} + \frac{u^2\tan\phi}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v$$

$$\frac{Dw}{Dt} + \qquad -2\Omega\cos\phi\ u \quad - \frac{u^2+v^2}{r} \qquad = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g - \nu\nabla^2 w$$

Typical “Synoptic Scales”



(from web.kma.go.kr)

Typical “Synoptic Scales”

<u>Typical “synoptic scales” in mid-latitudes</u>

$U \sim 10\ \mathrm{m\ s^{-1}}$	horizontal velocity scale
$W \sim 1\ \mathrm{cm\ s^{-1}}$	vertical velocity scale
$L \sim 10^6\ \mathrm{m}$	length scale [$\sim 1/(2\pi)$ wavelength]
$H \sim 10^4\ \mathrm{m}$	depth scale
$\delta P/\rho \sim 10^3\ \mathrm{m^2\ s^{-2}}$	horizontal pressure fluctuation scale
$L/U \sim 10^5\ \mathrm{s}$	time scale
$f \sim 10^{-4}\ \mathrm{s^{-1}}$	
$\nu \sim 10^{-5}\ \mathrm{m^2\ s^{-1}}$	(considering only molecular viscosity)
$\delta P \sim 10^3\ \mathrm{Pa}$	(~10 hPa)
$P \sim 10^5\ \mathrm{Pa}$	

(From Introduction to dynamic meteorology, Holton, 4th Ed.)

Momentum equation in vector form (with rotation)

$$\frac{Du}{Dt} - 2\Omega\sin\phi\ v + 2\Omega\cos\phi\ w + \frac{uw}{r} - \frac{uv\tan\phi}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\frac{Dv}{Dt} + 2\Omega\sin\phi\ u \qquad + \frac{vw}{r} + \frac{u^2\tan\phi}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v$$

$$\frac{Dw}{Dt} + \qquad -2\Omega\cos\phi\ u \quad - \frac{u^2+v^2}{r} \quad = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g - \nu\nabla^2 w$$

Momentum equation in vector form (with rotation)

$$\frac{D\vec{\mathbf{v}}}{Dt} = -2\vec{\mathbf{\Omega}}\times\vec{\mathbf{v}} - \frac{1}{\rho}\nabla p - \mathbf{k}g + \nu\nabla^2\vec{\mathbf{v}}$$

where $\vec{\mathbf{v}} = \mathbf{i}u + \mathbf{j}v + \mathbf{k}w$

Table 2.1 *Scale Analysis of the Horizontal Momentum Equations*

	A	B	C	D	E	F	G
$x-$Eq.	$\frac{Du}{Dt}$	$-2\Omega v\sin\phi$	$+2\Omega w\cos\phi$	$+\frac{uw}{a}$	$-\frac{uv\tan\phi}{a}$	$=-\frac{1}{\rho}\frac{\partial p}{\partial x}$	$+F_{rx}$
$y-$Eq.	$\frac{Dv}{Dt}$	$+2\Omega u\sin\phi$		$+\frac{vw}{a}$	$+\frac{u^2\tan\phi}{a}$	$=-\frac{1}{\rho}\frac{\partial p}{\partial y}$	$+F_{ry}$
Scales	U^2/L	f_0U	f_0W	$\frac{UW}{a}$	$\frac{U^2}{a}$	$\frac{\delta P}{\rho L}$	$\frac{\nu U}{H^2}$
(m s^{-2})	10^{-4}	10^{-3}	10^{-6}	10^{-8}	10^{-5}	10^{-3}	10^{-12}

Table 2.2 *Scale Analysis of the Vertical Momentum Equation*

z - Eq.	Dw/Dt	$-2\Omega u\cos\phi$	$-(u^2+v^2)/a$	$=-\rho^{-1}\partial p/\partial z$	$-g$	$+F_{rz}$
Scales	UW/L	f_0U	U^2/a	$P_0/(\rho H)$	g	νWH^{-2}
m s^{-2}	10^{-7}	10^{-3}	10^{-5}	10	10	10^{-15}

(From Introduction to dynamic meteorology, Holton, 4th Ed.)

Primitive equations

Approximations

1. Hydrostatic approximation

   Consider only the major balance in z $: \partial p / \partial z = -\rho g$

2. Shallow fluid approximation

   Let $r = a + z$ *and assume* $r \to a$ *(thus* $1/r^2\, \partial(r^2 w)/\partial z \to \partial w / \partial z$*)*

3. Traditional approximation

   Coriolis force involving 'w*', small metric terms (*$uw/r\ldots$*) are neglected*

$$\frac{Du}{Dt} - 2\Omega v \sin\phi - \frac{uv\tan\phi}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\frac{Dv}{Dt} + 2\Omega u \sin\phi + \frac{u^2\tan\phi}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$