Vector Analysis

1. Basic concept of vector
2. Coordinate and unit vector
3. Differentiation of vector
4. Del (∇) operator

Vector

- A quantity having **magnitude** and **direction**
- Made of two or more scalar values (e.g., $\vec{v} = (u, v, w)$)

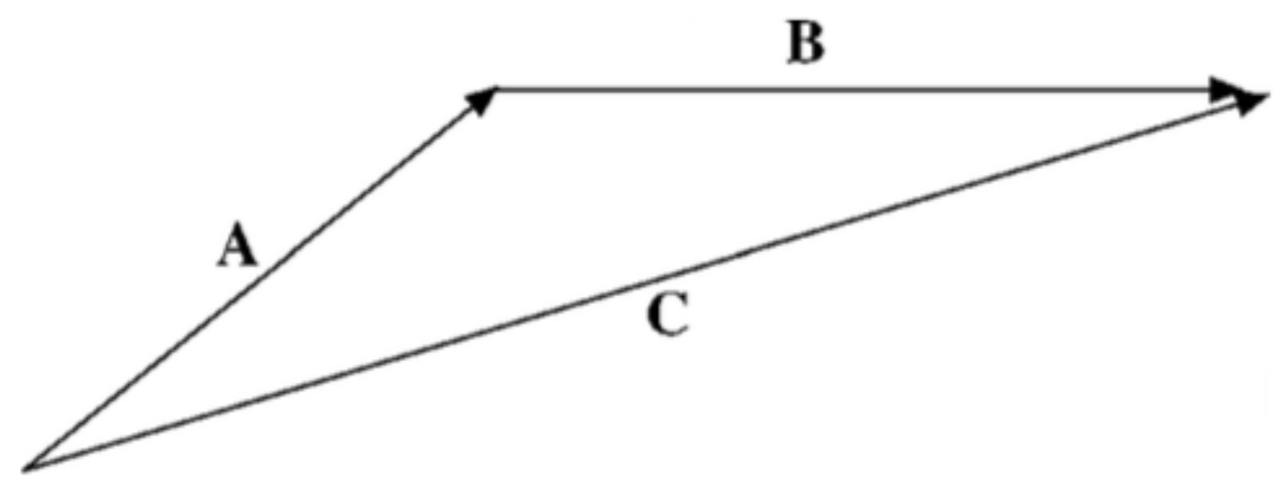


FIGURE 1.1 Triangle law of vector addition.

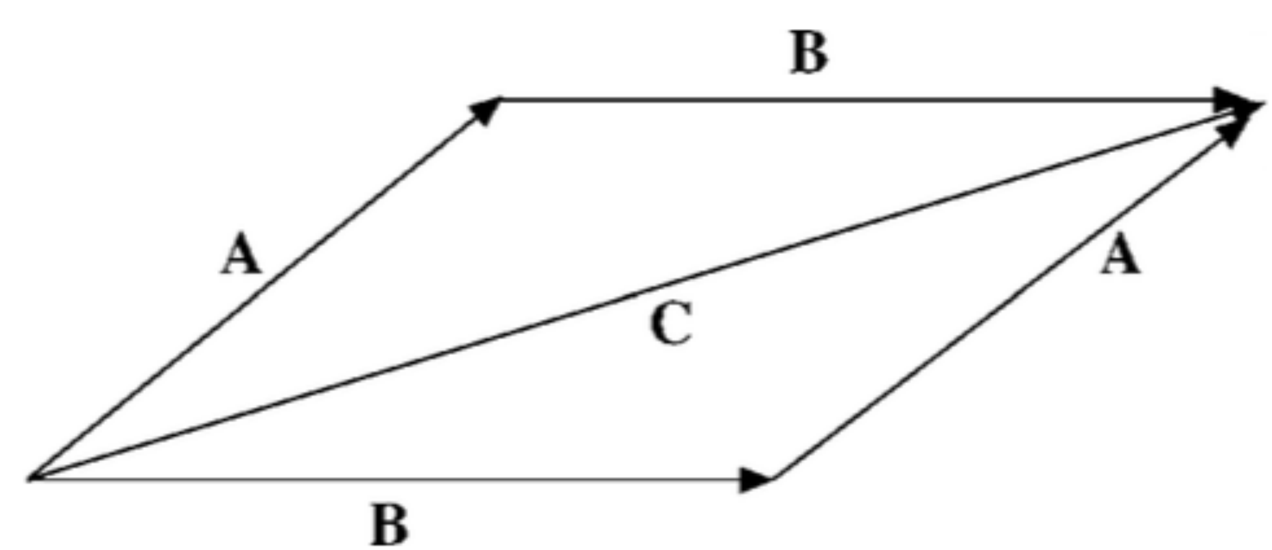


FIGURE 1.2 Parallelogram law of vector addition.

Vector

- A quant
  magnit
- Made o
  scalar v
  (e.g., $\vec{v}$

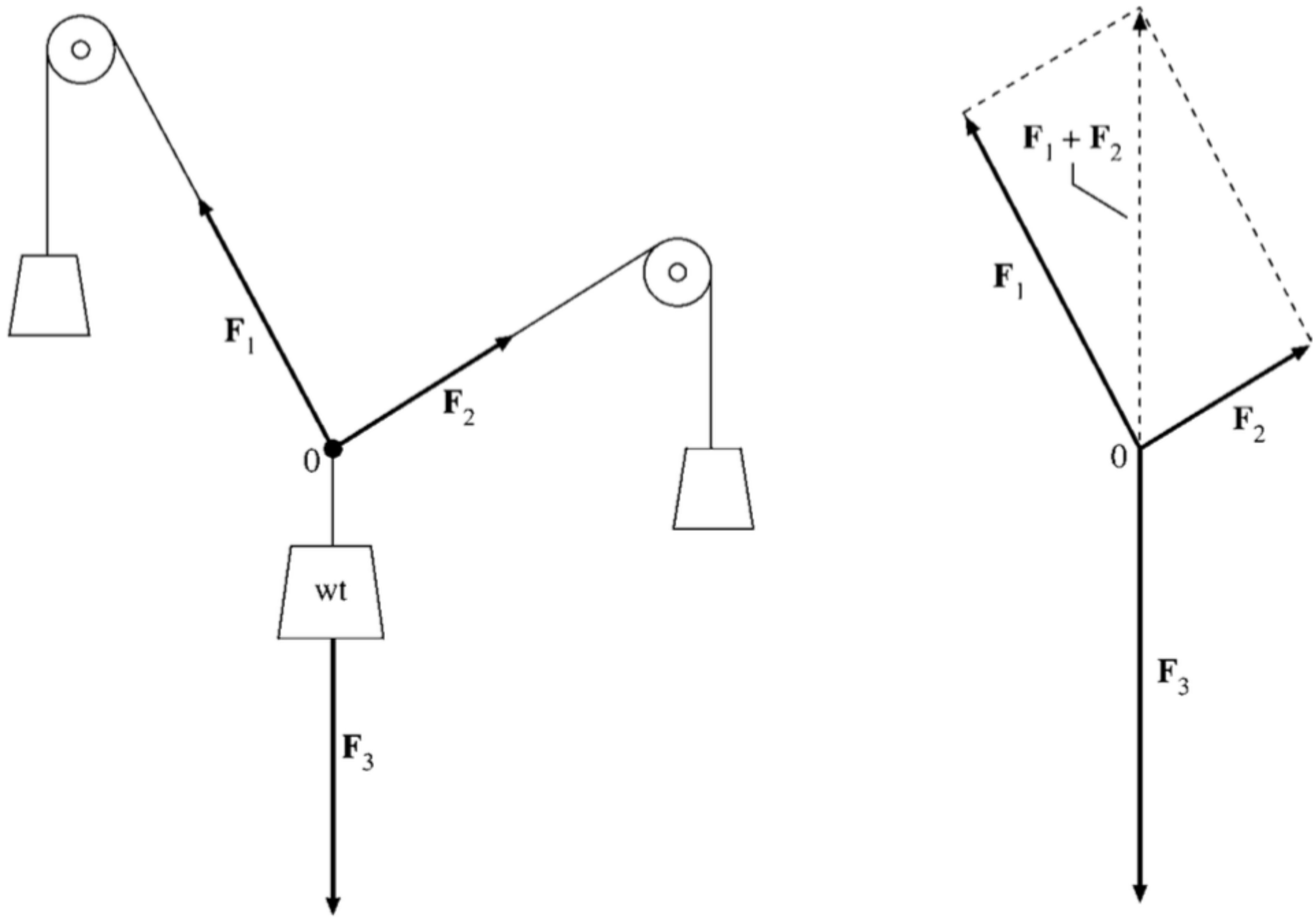
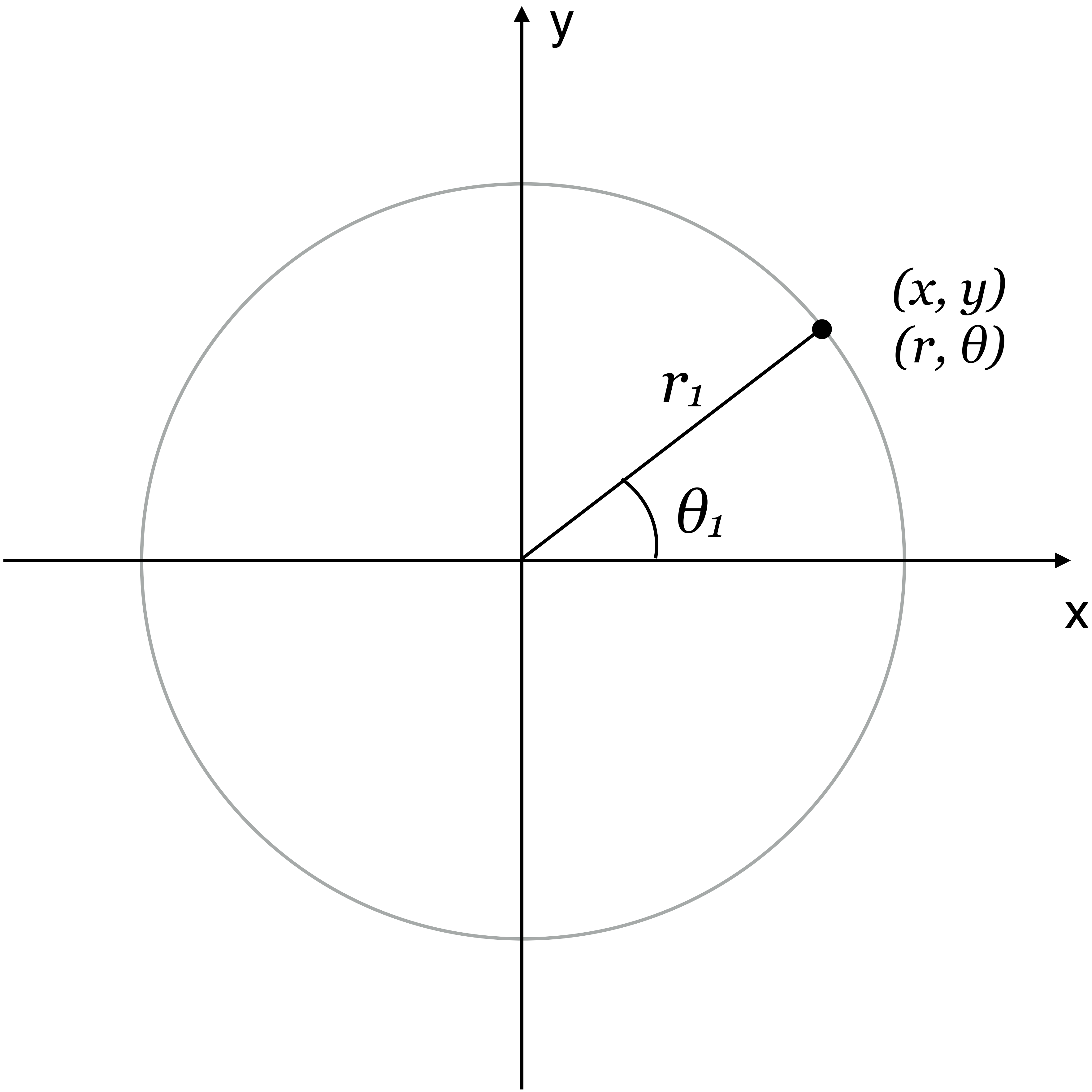


FIGURE 1.4 Equilibrium of forces: $\mathbf{F}_1 + \mathbf{F}_2 = -\mathbf{F}_3$.

Cartesian coordinate



$$z = x + i\,y$$

$$z = r\,e^{i\,\theta}$$

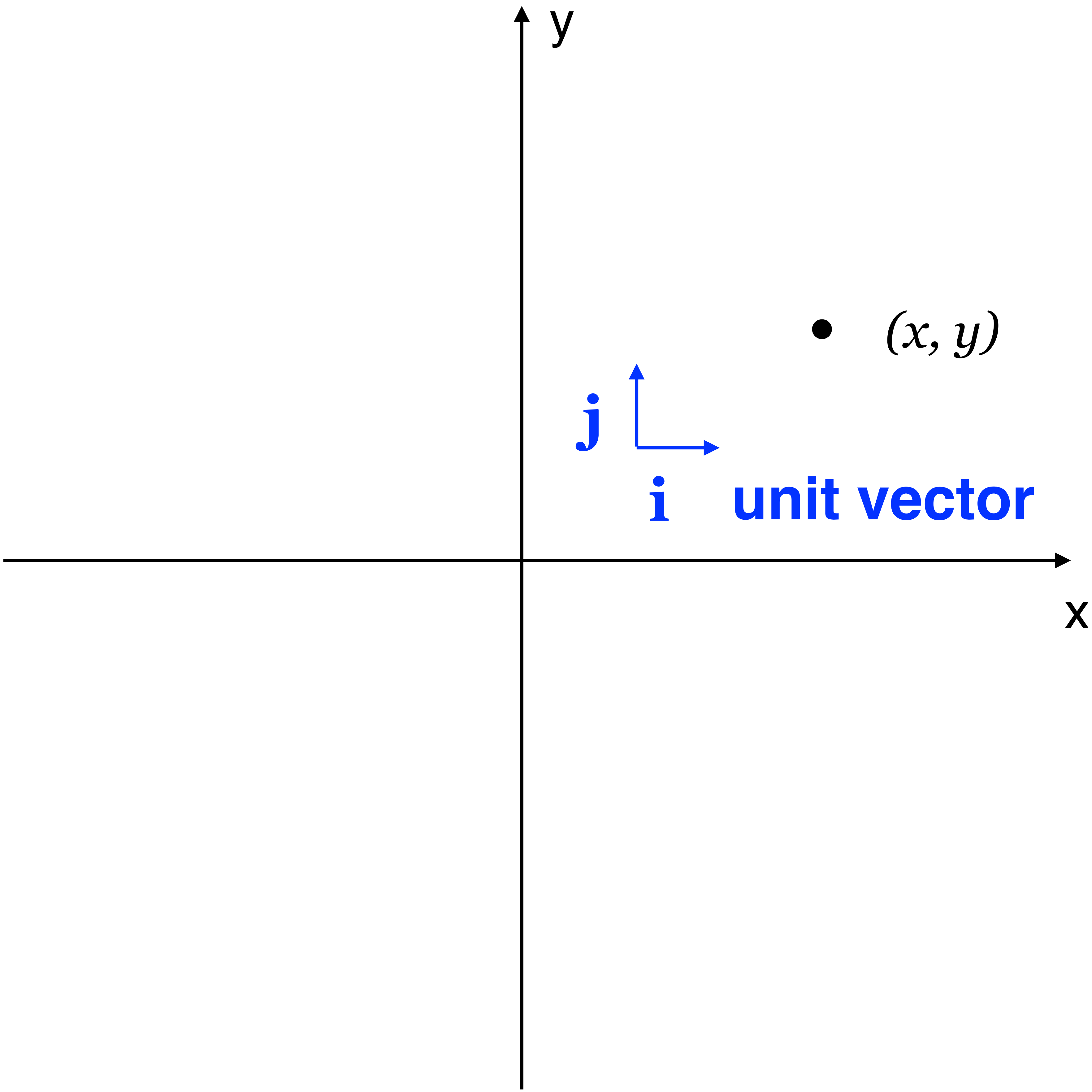
$x = r \cos \theta$

$y = r \sin \theta$

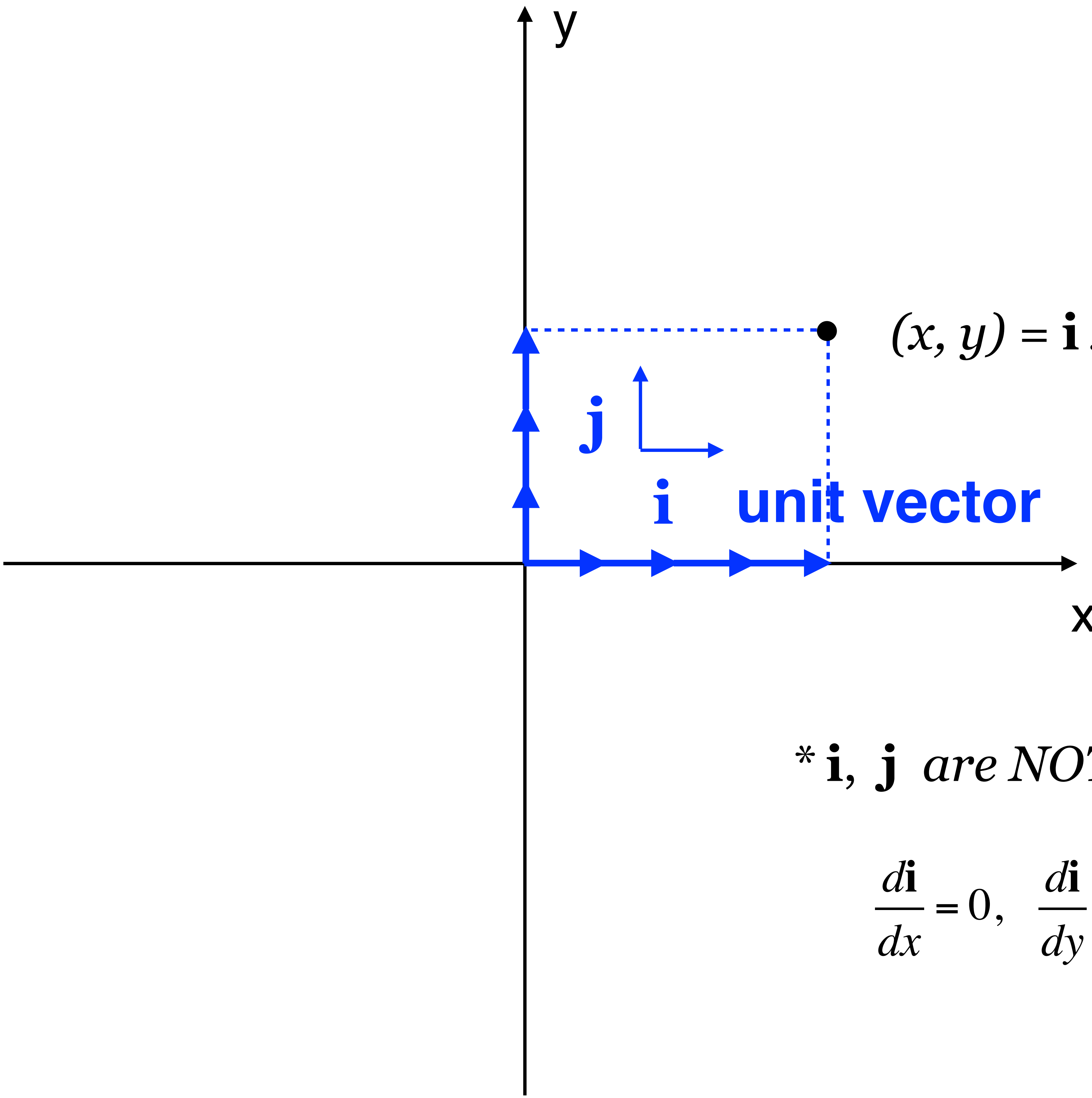
** recall Euler's formula*

$e^{i\theta} = \cos \theta + i \sin\theta$

Cartesian coordinate



Cartesian coordinate



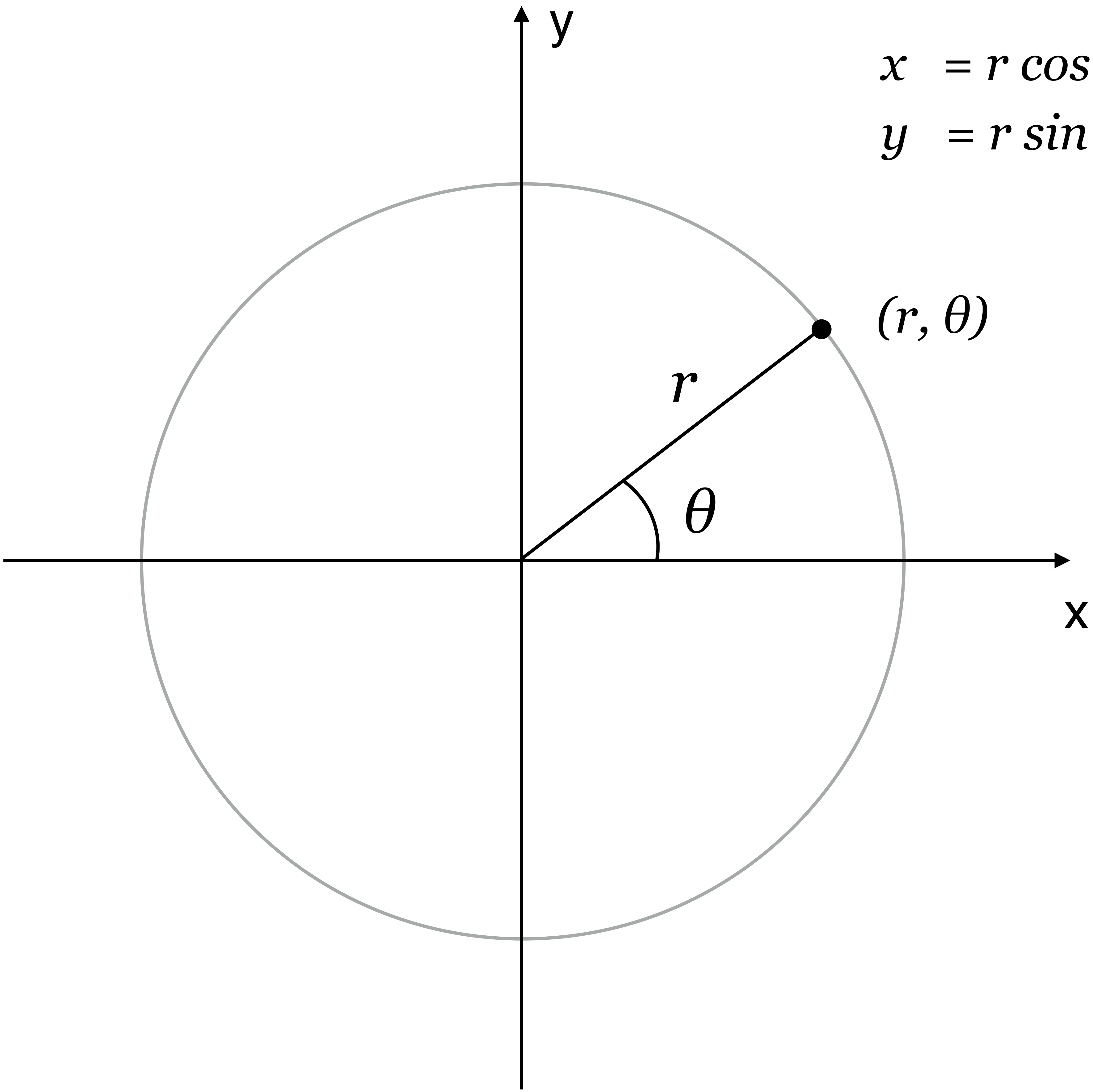
$$(x, y) = \mathbf{i}\,x + \mathbf{j}\,y$$

* $\mathbf{i},\ \mathbf{j}$ *are NOT varying with position*

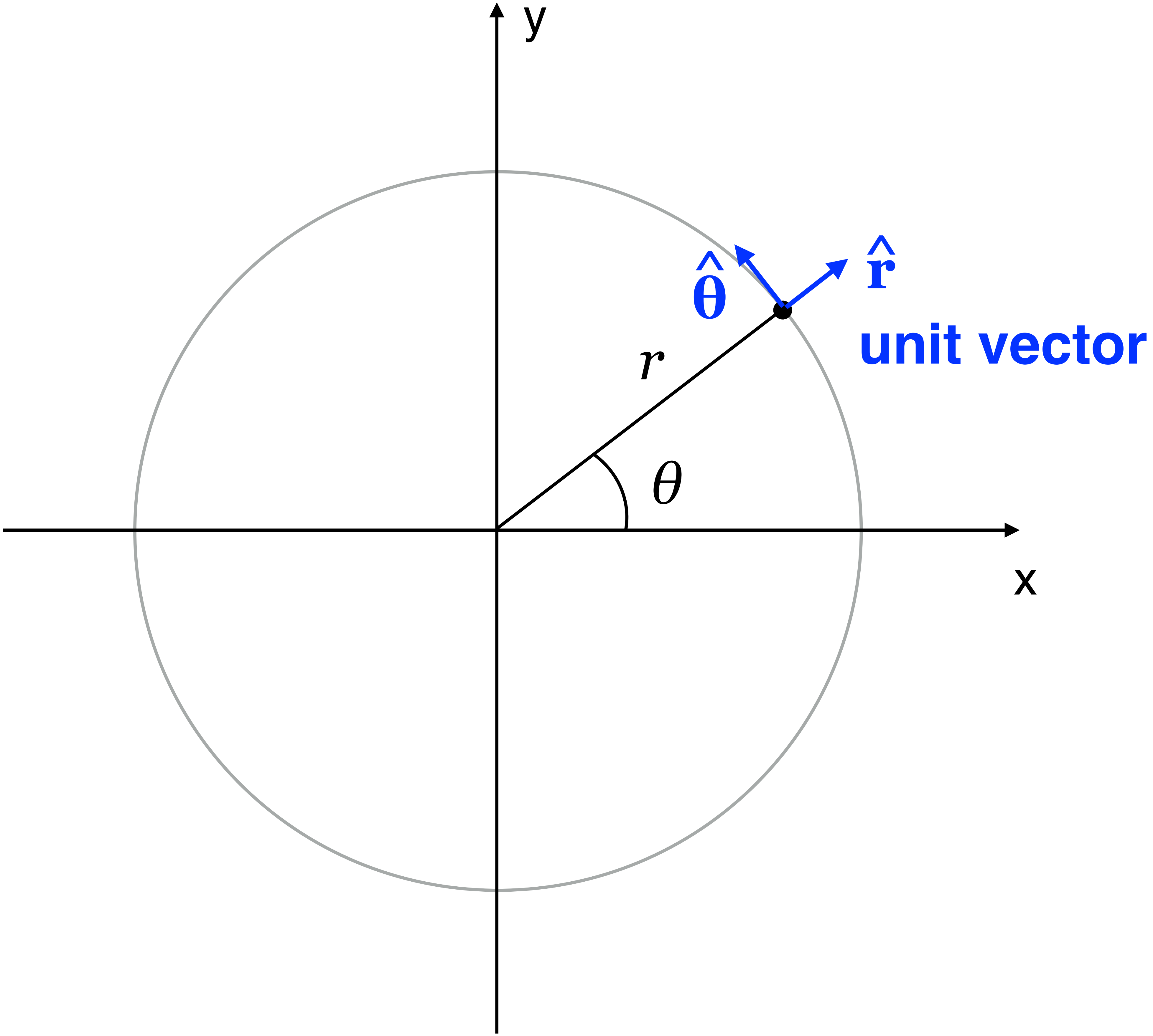
$$\frac{d\mathbf{i}}{dx} = 0, \quad \frac{d\mathbf{i}}{dy} = 0, \quad \frac{d\mathbf{j}}{dx} = 0, \quad \frac{d\mathbf{j}}{dy} = 0$$

Polar coordinate

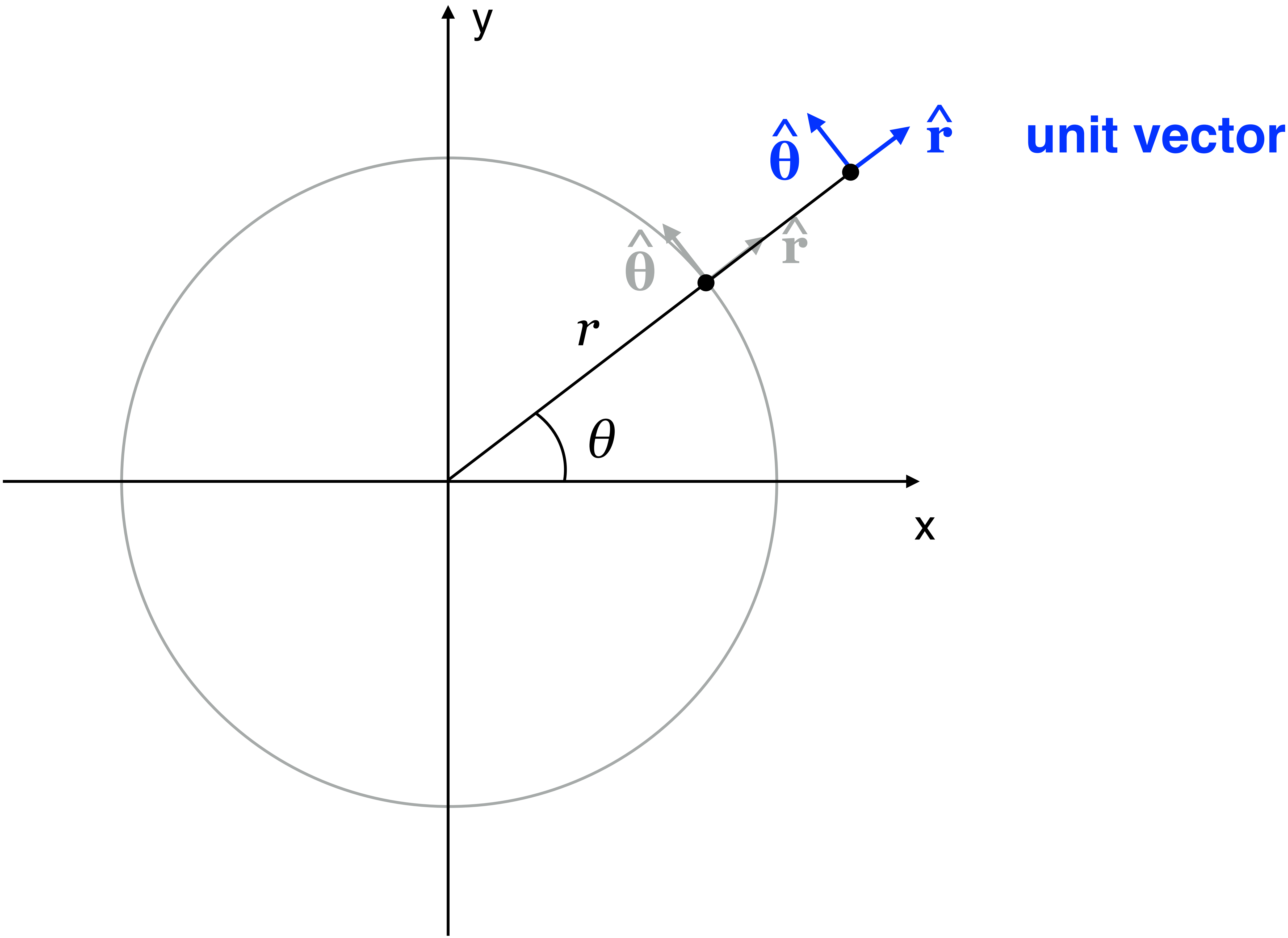
$$x = r\cos\theta$$
$$y = r\sin\theta$$



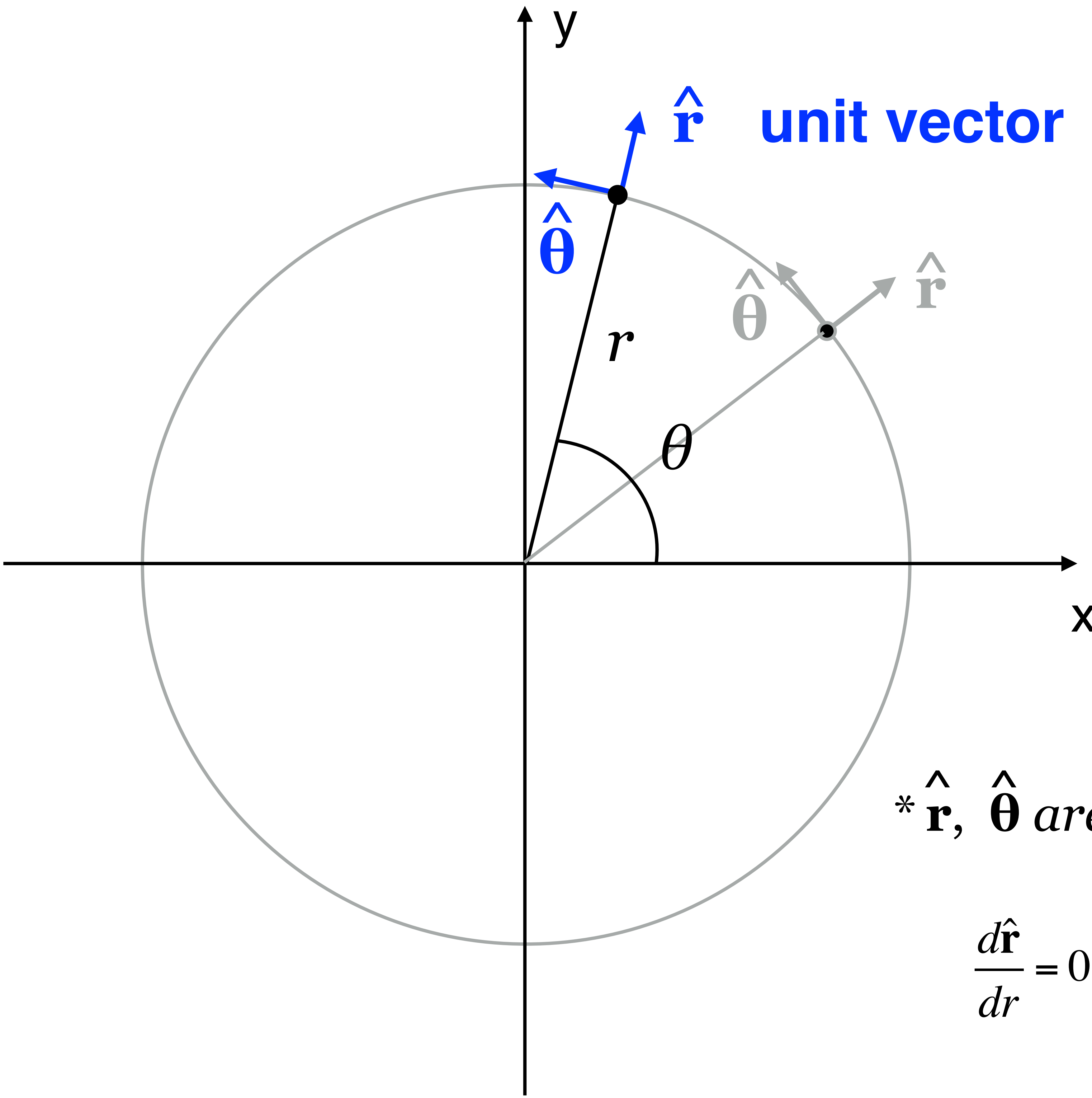
Polar coordinate



Polar coordinate



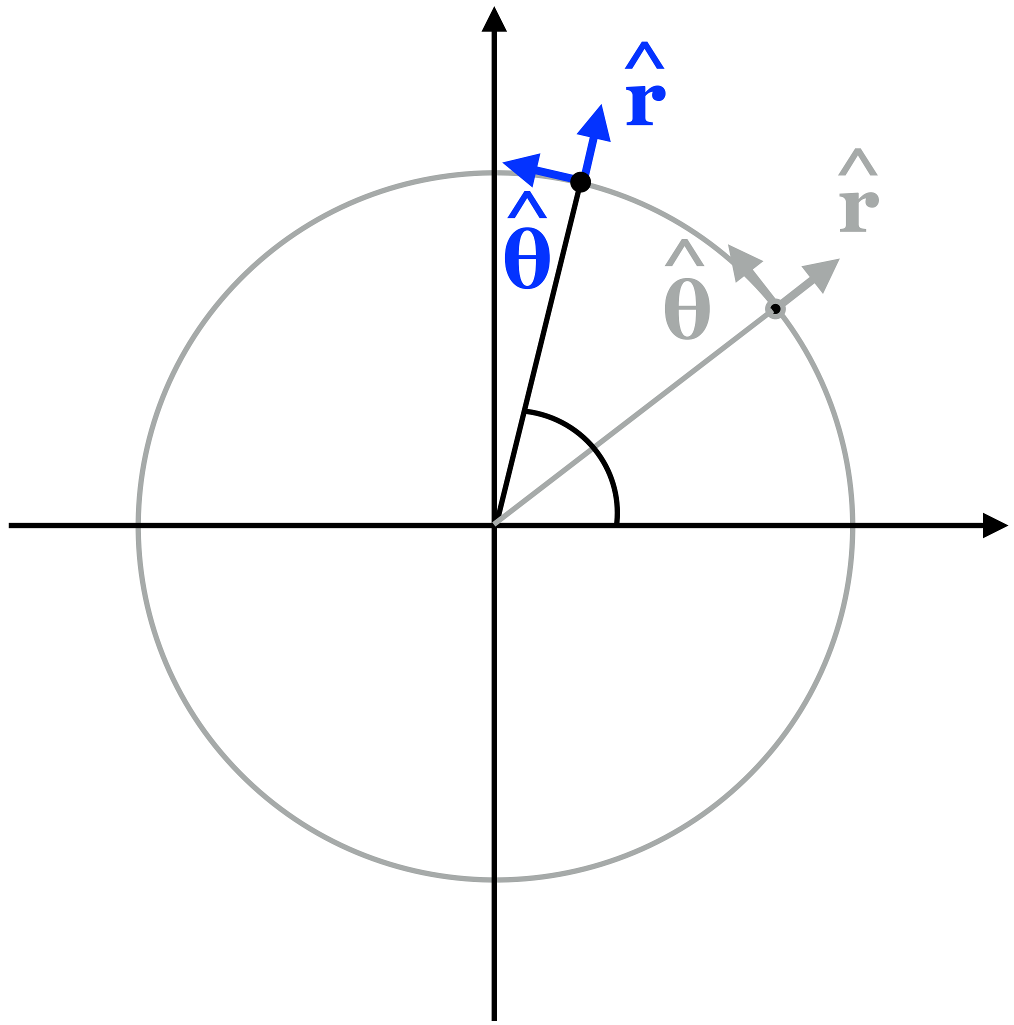
Polar coordinate

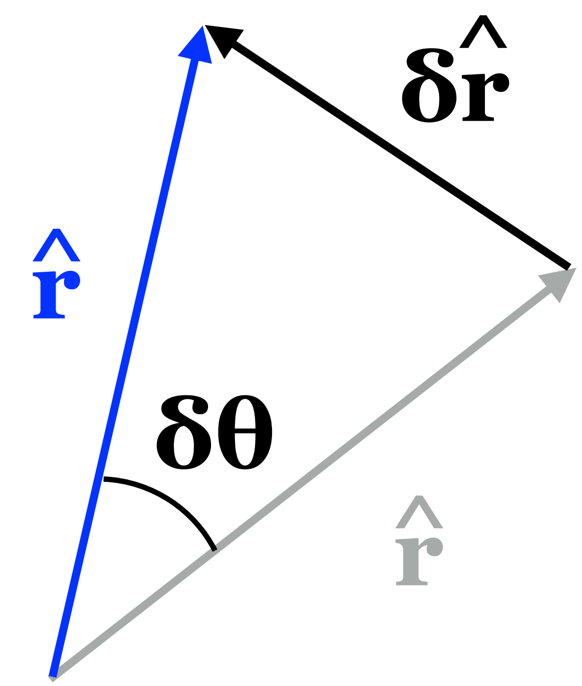


* $\hat{\mathbf{r}}$, $\hat{\boldsymbol{\theta}}$ *are varying with position*

$$\frac{d\hat{\mathbf{r}}}{dr} = 0, \quad \frac{d\hat{\mathbf{r}}}{d\theta} \neq 0, \quad \frac{d\hat{\theta}}{dr} = 0, \quad \frac{d\hat{\theta}}{d\theta} \neq 0$$

Polar coordinate

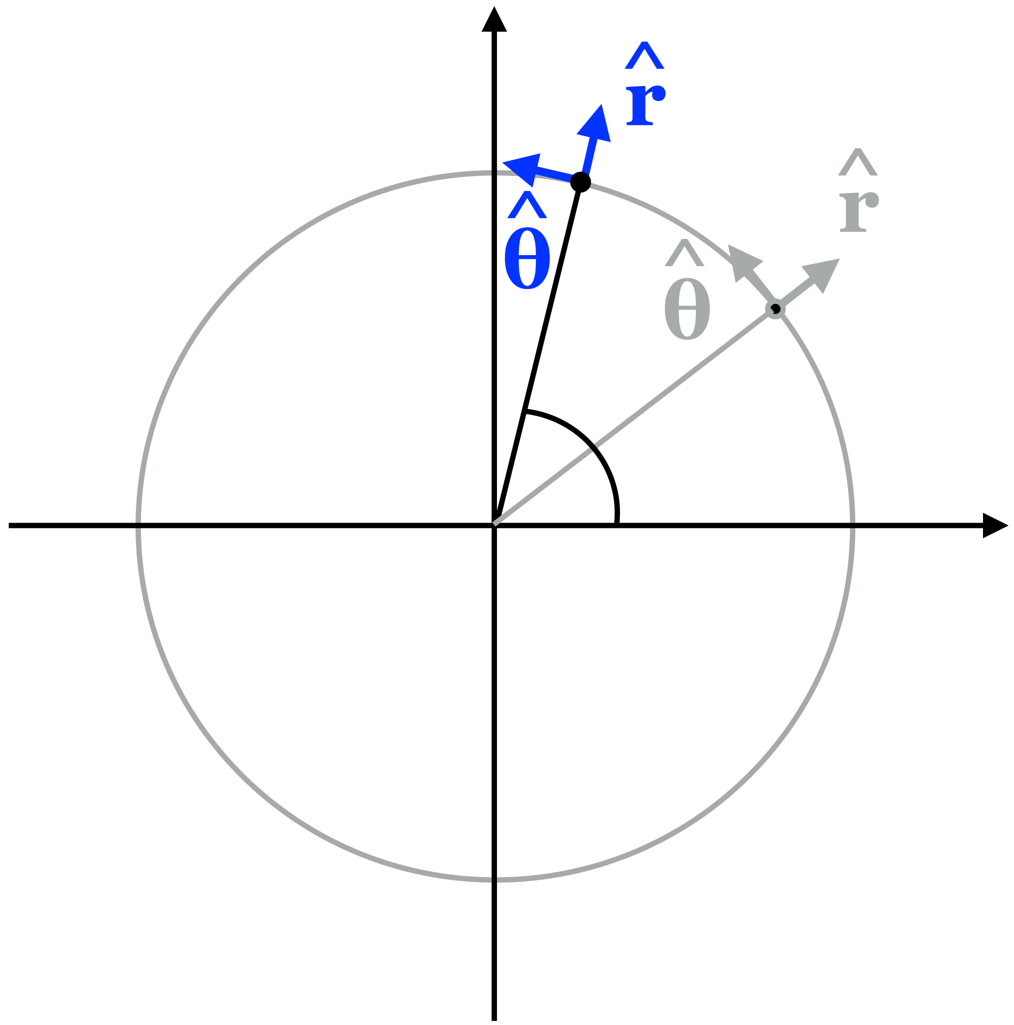


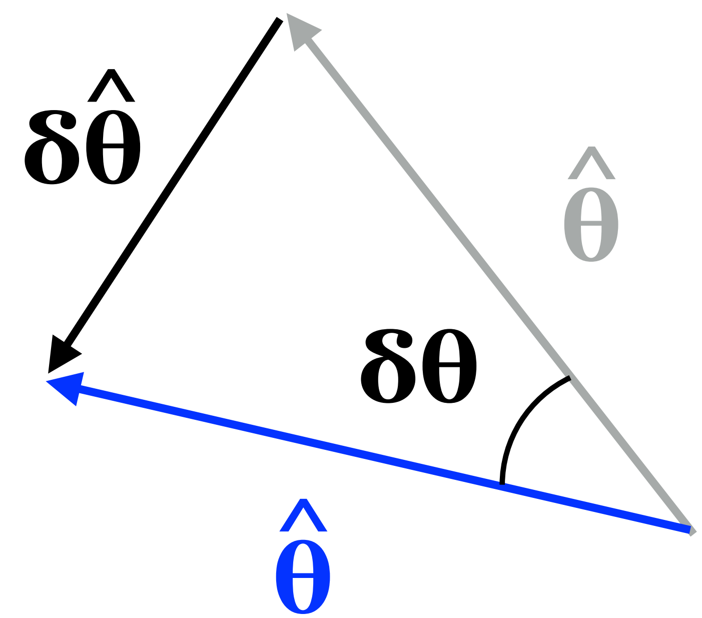


* $\hat{\mathbf{r}}$, $\hat{\boldsymbol{\theta}}$ *are varying with "θ"*

$$\lim_{\delta\theta \to 0} \frac{\delta \hat{\mathbf{r}}}{\delta\theta} = \frac{d\hat{\mathbf{r}}}{d\theta} = \hat{\theta}$$

Polar coordinate





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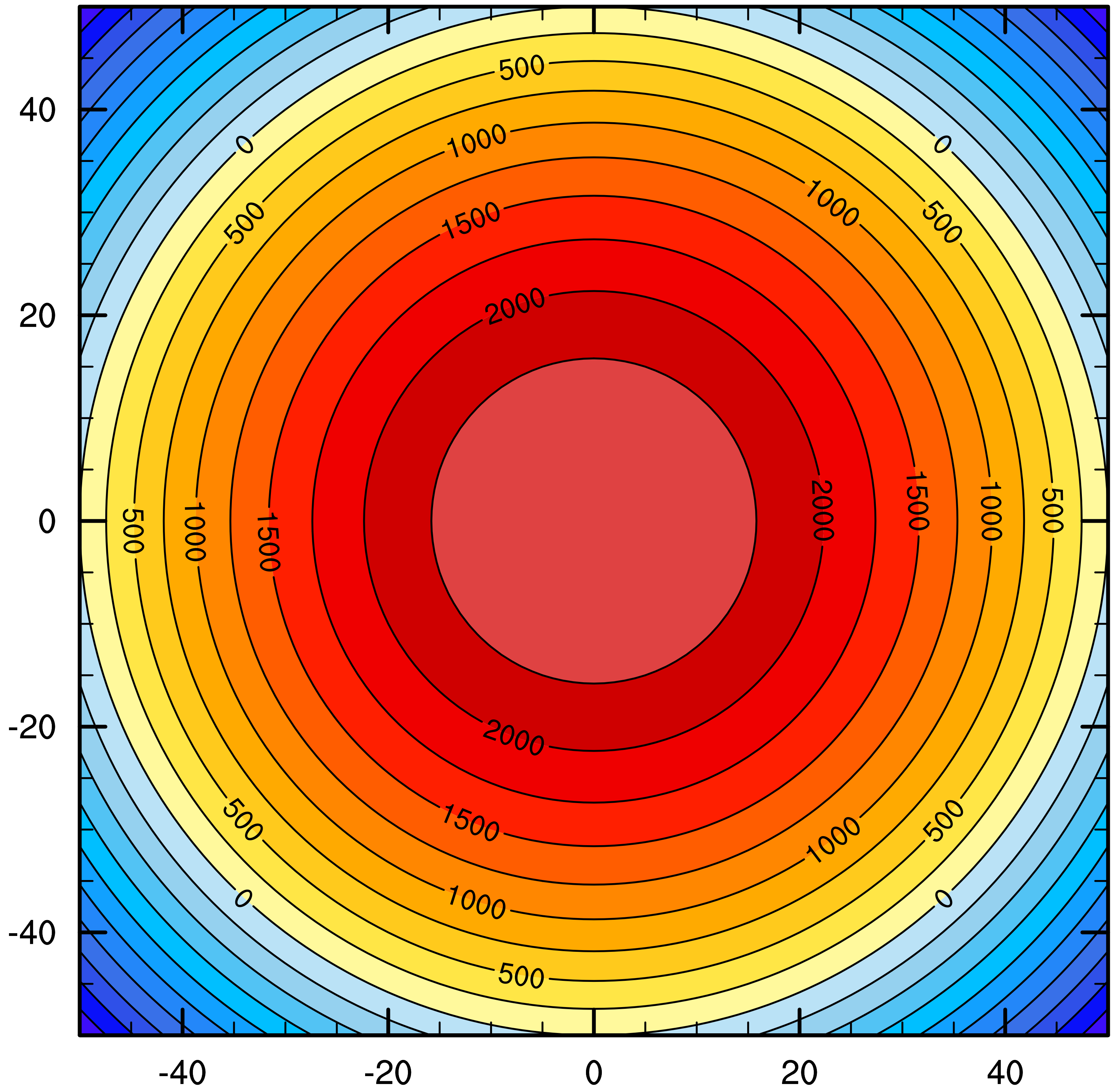
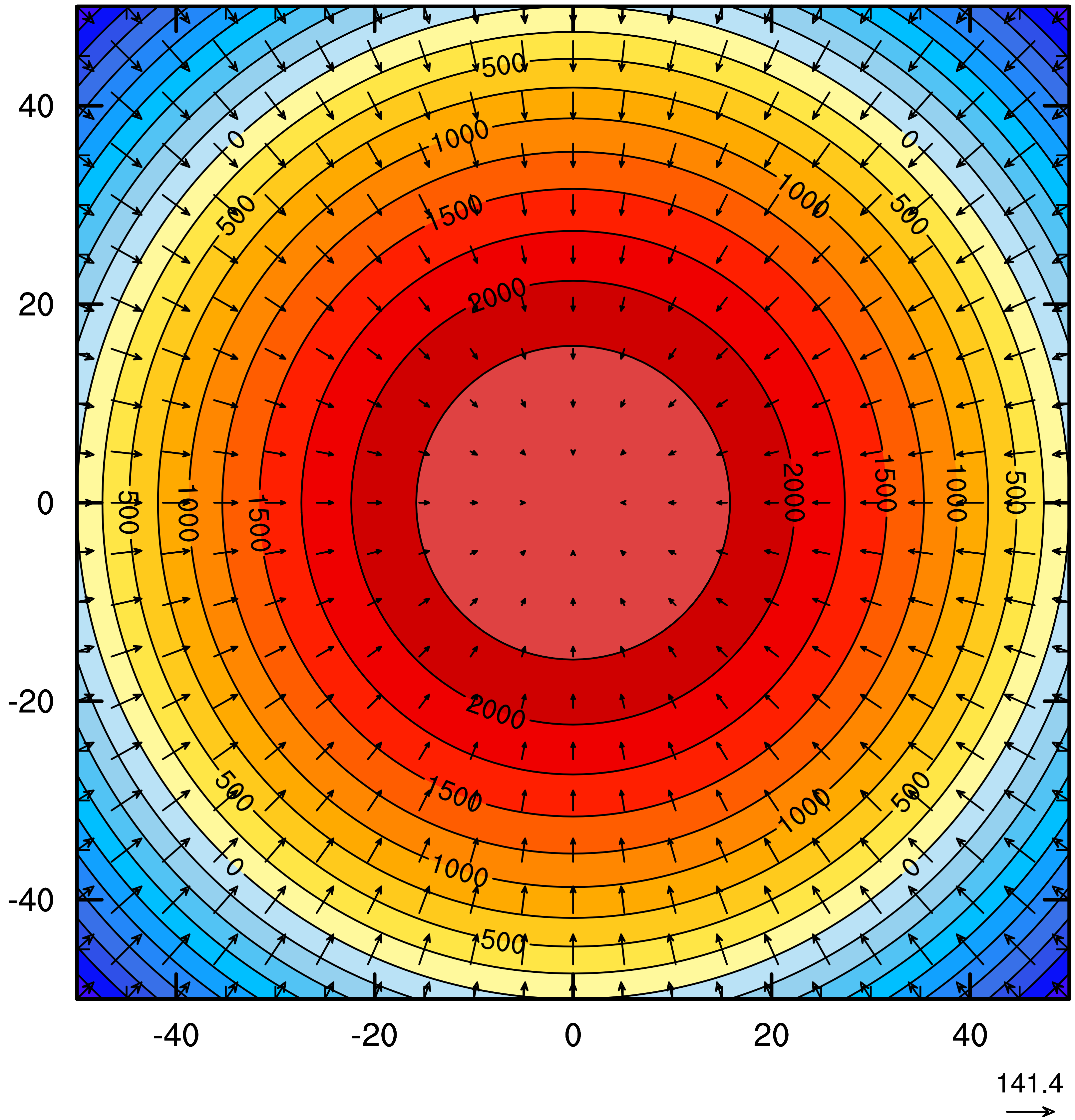
$$\lim_{\delta\theta \to 0} \frac{\delta \hat{\theta}}{\delta\theta} = \frac{d\hat{\theta}}{d\theta} = \hat{\mathrm{r}}$$

Gradient ($\nabla\Phi$)



$$\Phi = 2500 - x^2 - y^2$$

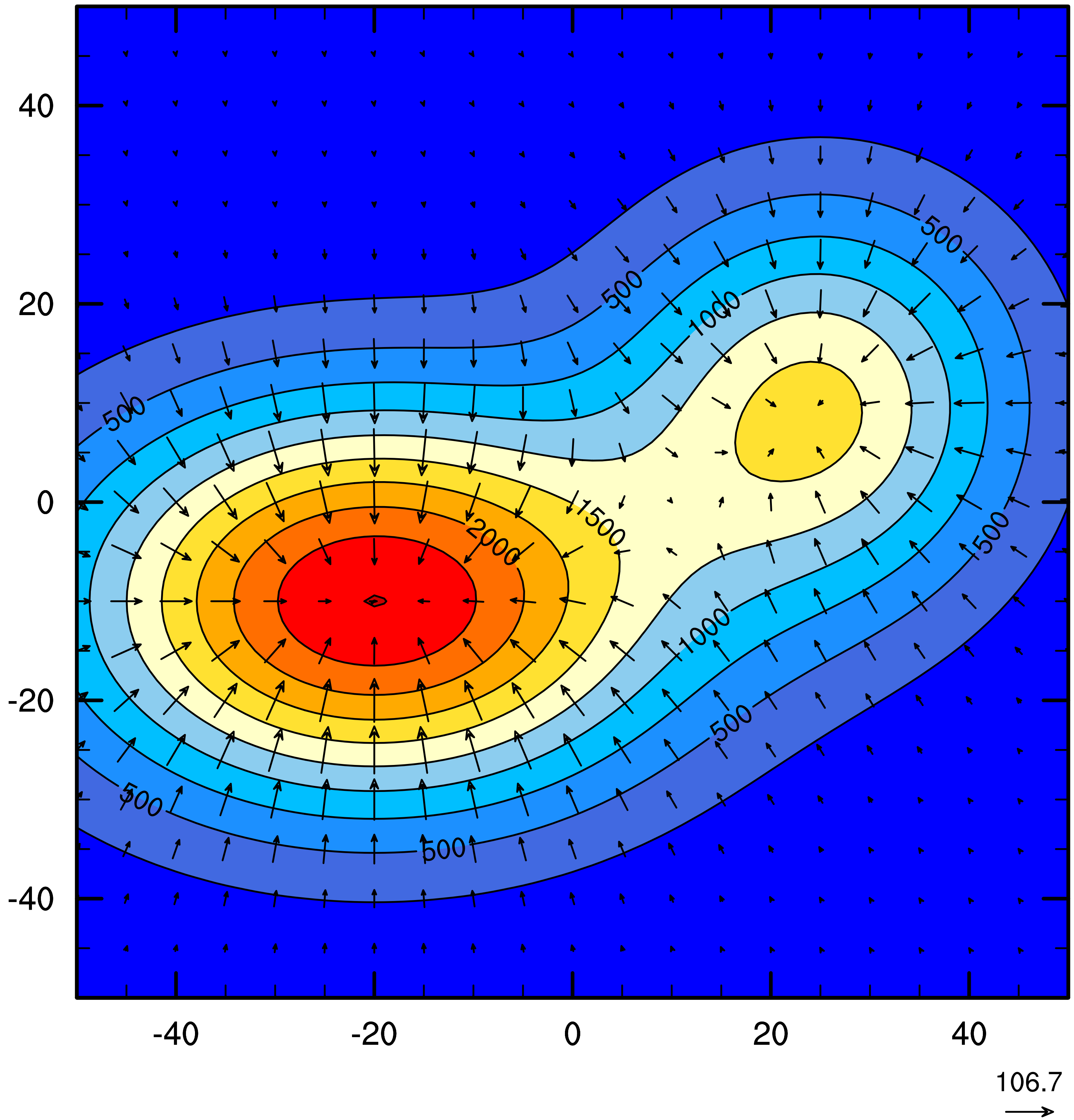
Gradient ($\nabla\Phi$)



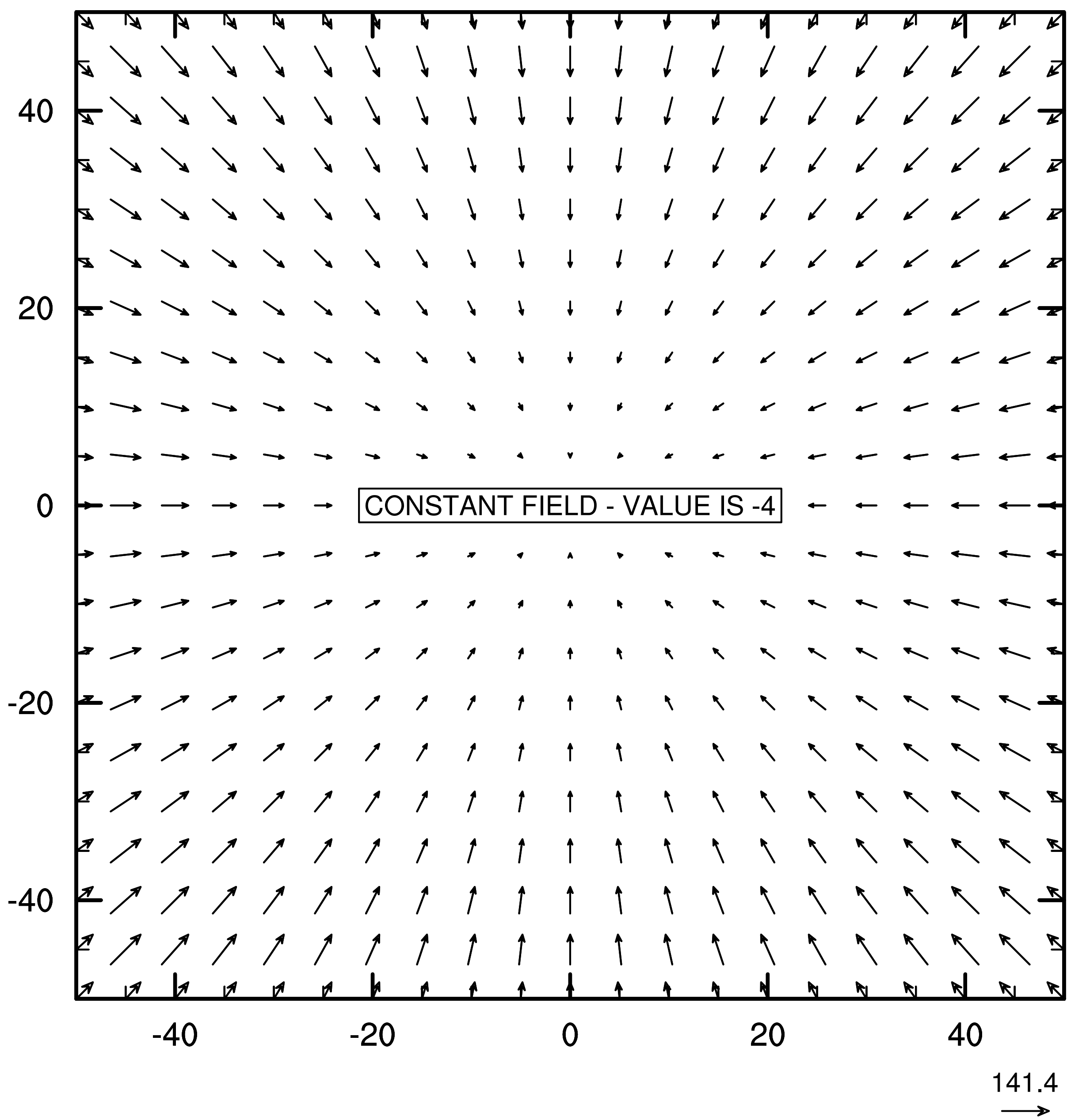
$$\Phi = 2500 - x^2 - y^2$$

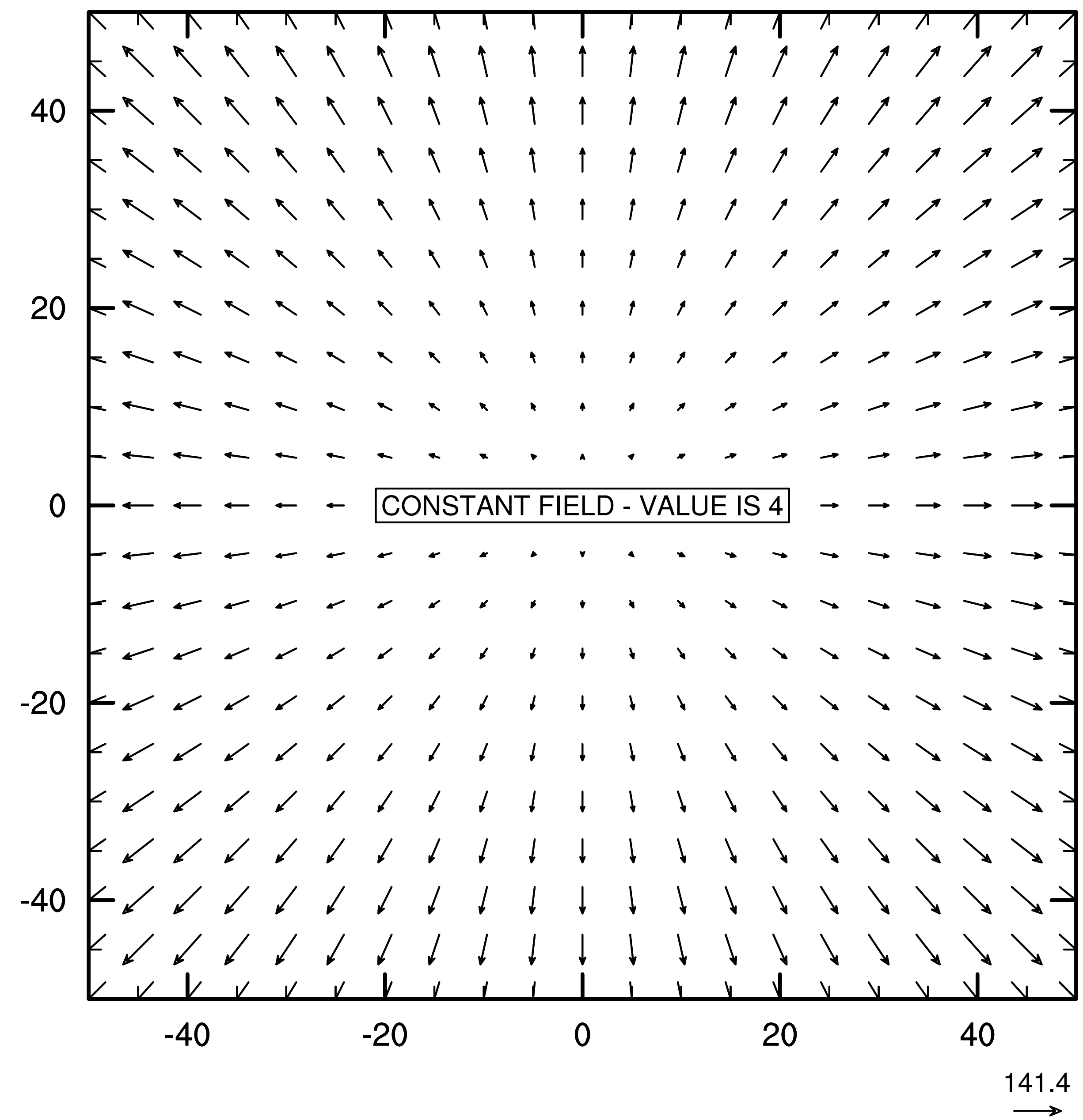
$$\nabla\Phi = -\mathbf{i}2x - \mathbf{j}2y$$
$$= (-2x,\ -2y)$$

Gradient ($\nabla\Phi$)



Divergence ($\nabla \bullet \vec{V}$)





Curl ($\nabla \times \vec{V}$)