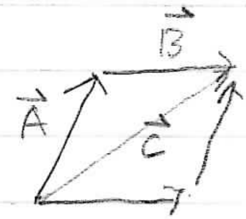
Basic math
Vector 1

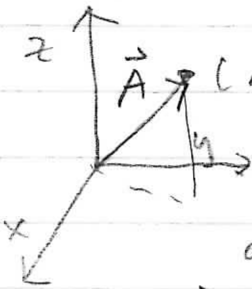
Basic properties of vector

$$\vec{C} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$



$$a(\vec{A} + \vec{B}) = a\vec{A} + a\vec{B}$$
$$(a+b)\vec{A} = a\vec{A} + b\vec{A}$$

- expression in coordinate system

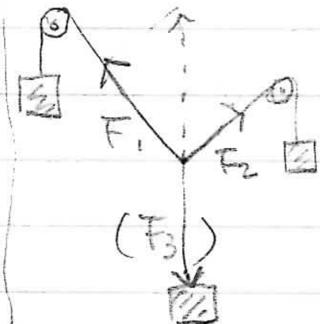


$$(A_x, A_y, A_z) = \vec{A}$$

or

$$\vec{A} = i A_x + j A_y + k A_z$$

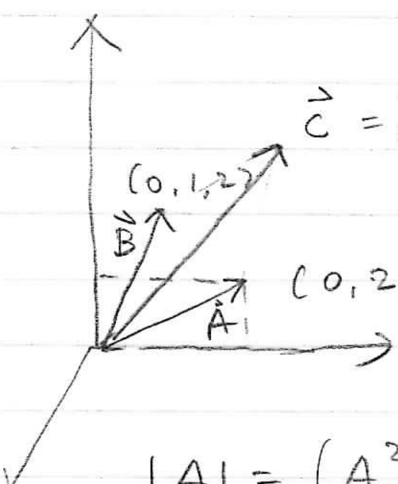
i, j, k are orthonormal unit vectors



$$\underline{F_1 + F_2} + F_3 = 0$$
$$F_3 = -(F_1 + F_2)$$

$$\vec{A} + \vec{B} = (A_x, A_y, A_z) + (B_x, B_y, B_z)$$
$$= (A_x + B_x,\ A_y + B_y,\ A_z + B_z)$$

$$\vec{A} + \vec{B} = i(A_x + B_x) + j(A_y + B_y) + k(A_z + B_z)$$



$$\vec{C} = \vec{A} + \vec{B} = (0,3,3)$$

Benefit of coordinate expression (just add numbers!)

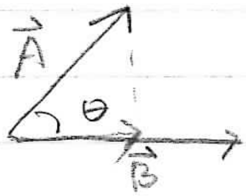
$$|A| = (A_x^2 + A_y^2 + A_z^2)^{1/2}$$ Pitagorian theorem.

$$|C| = (0 + 9 + 9)^{1/2} = \sqrt{18} = 3\sqrt{2}$$

Basic math

Vector 2

Dot product (내적) ← 동일한 성분을 찾아내는 방법



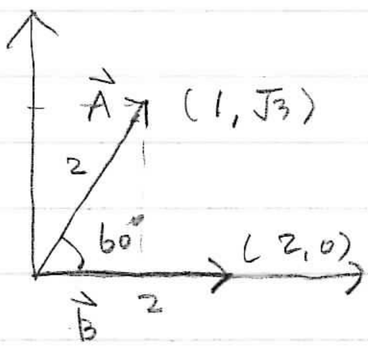
$$\vec{A} \cdot \vec{B} = |A||B| \cos\theta$$

In cartesian coordinate

$$(A_x, A_y, A_z) \cdot (B_x, B_y, B_z) = A_x B_x + A_y B_y + A_z B_z$$

$$i \cdot i = 1 \ ; \ i \cdot j = 0 \ ; \ i \cdot k = 0 \ ; \ j \cdot k = 0$$

$$(i A_x + j A_y + k A_z) \cdot i = A_x$$ ← extract i component.



$$\vec{A} \cdot \vec{B} = 2 \cdot 2 \cdot \cos 60° = \underline{2}$$

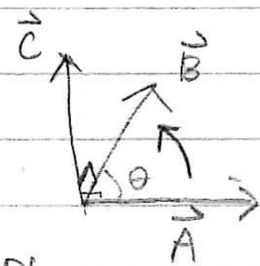
$$(2, 0) \cdot (1, \sqrt{3}) = 2 + 0 \cdot \sqrt{3} = \underline{2}$$

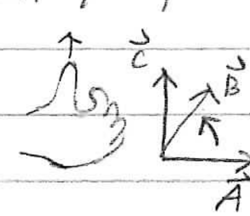
Cross product (외적) 1/2

$$\vec{C} = \vec{A} \times \vec{B}$$

$|C| = |A| \cdot |B| \sin\theta$, $\vec{C} \perp \vec{A}$; $\vec{C} \perp \vec{B}$

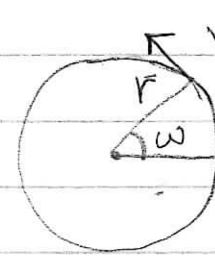
(direction, right-handed system)





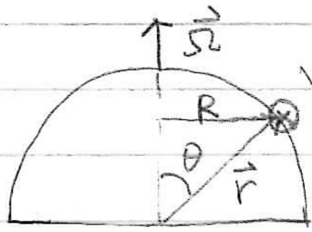
▶ <u>Physics</u>

- Angular velocity
- Angular momentum
- Torque



simple

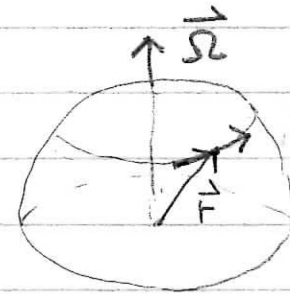
but reality



$v = \Omega R$
$= \underline{\Omega r \cdot \sin\theta}$

$\boxed{\text{AREA}}$

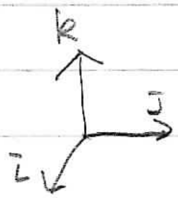
$\Leftarrow$



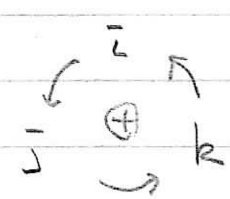
$\vec{V} = \vec{\Omega} \, ? \, \vec{r}$
$= \underline{\vec{\Omega} \times \vec{r}}$

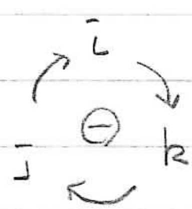
can be genalize for any vector $\vec{r} \Rightarrow \vec{u}$ ...

▶ Mathematics



$i \times j = k$ $\quad$ $j \times i = -k$
$j \times k = i$ $\quad$ $k \times j = -i$
$k \times i = j$ $\quad$ $i \times k = -j$

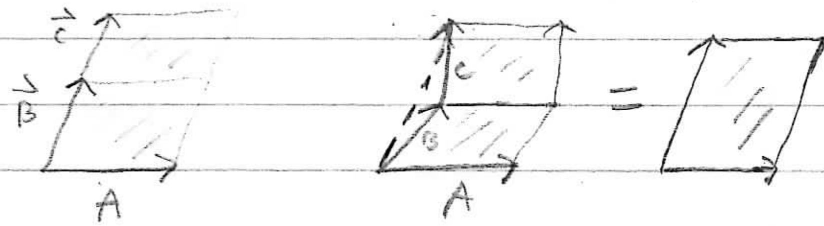




Basic math
vector 4

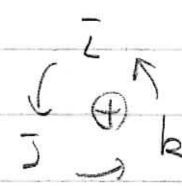
Cross product (외적) 2/2

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$



$$\begin{aligned}
\vec{A} \times \vec{B} &= \vec{A} \times (\bar{i}B_x + \bar{j}B_y + kB_z) \\
&= (\bar{i}A_x + \bar{j}A_y + kA_z) \times \bar{i}B_x \\
&\quad + (\bar{i}A_x + \bar{j}A_y + kA_z) \times \bar{j}B_y \\
&\quad + (\bar{i}A_x + \bar{j}A_y + kA_z) \times kB_z \\
&= \bar{j} \times k(-A_zB_y + A_yB_z) \\
&\quad + k \times \bar{i}(A_zB_x - A_xB_z) \\
&\quad + \bar{i} \times \bar{j}(-A_yB_x + A_xB_y)
\end{aligned}$$

$$\begin{aligned}
\vec{A} \times \vec{B} = \; &\bar{i}(A_yB_z - A_zB_y) \\
&+ \bar{j}(A_zB_x - A_xB_z) \\
&+ k(A_xB_y - A_yB_x)
\end{aligned}$$



Del operator

Total variation of a function $F(x(t), y(t), z(t))$ during small tim δt

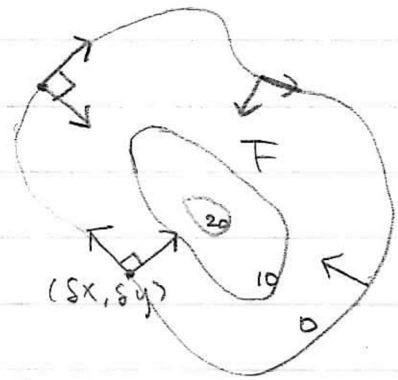
$$\frac{dF}{dt} = \frac{\partial F}{\partial x}\underbrace{\frac{dx}{dt}}_{u} + \frac{\partial F}{\partial y}\underbrace{\frac{dy}{dt}}_{v} + \frac{\partial F}{\partial z}\underbrace{\frac{dz}{dt}}_{w}$$

$$= \left(i\frac{\partial F}{\partial x} + j\frac{\partial F}{\partial y} + k\frac{\partial F}{\partial z}\right)\cdot\left(iu + jv + kw\right)$$

$$= \nabla F \cdot \vec{V}$$

$$\boxed{\nabla \equiv i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}}$$

Gradient ∇F



$$\delta F = \frac{\partial F}{\partial x}\delta x + \frac{\partial F}{\partial y}\delta y$$

$$\left(i\frac{\partial F}{\partial x} + j\frac{\partial F}{\partial y}\right)\cdot\left(i\delta x + j\delta y\right) = 0$$

" ∇F is always orthogral to a constant contour line. "
& pointing to the direction obtaining maximum increase in F

Basic math
del operator 2

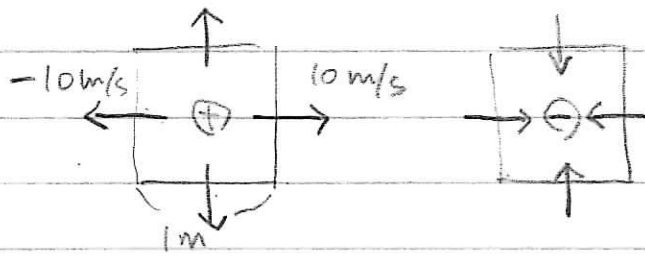
Divergence $\nabla \cdot \vec{V}$

$$\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \right) \cdot (u, v, w)$$

$$= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

A physical interpretation

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0}$$

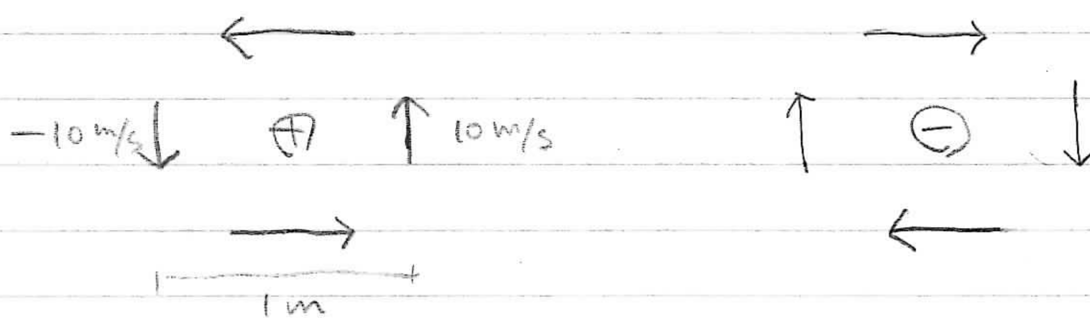


$$\frac{10\ \mathrm{m/s} - (-10\ \mathrm{m/s})}{1\ \mathrm{m}} = 20\ \mathrm{s}^{-1}$$

Curl $\nabla \times \vec{V}$

$$\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \right) \times (iu + jv + kw)$$

$$= i\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) + j\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) + k\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$



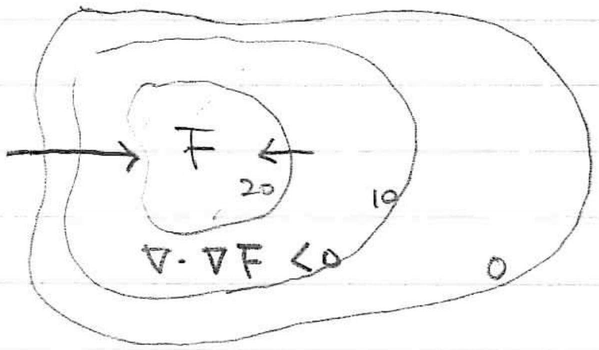
$$\frac{10\ \mathrm{m/s} - (-10\ \mathrm{m/s})}{1\ \mathrm{m}} = 20\ \mathrm{s}^{-1}$$

Basic math
del operator 3

Laplacian $\nabla^2 F$ ($= \nabla \cdot \nabla F$)

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) F$$

physically diffusion



lose a physical value
(F; e.g. Temperature, wind speed)
from the maxium region

e.g.) diffusion equation

$$\frac{\partial T}{\partial t} = c \nabla^2 T$$